Eksamen R1
Høst 2020
Løsningsforslag

DEL 1

Oppgave 1 (5 poeng)

a)

$$\begin{aligned} f(x) &= x^3 + e^x \\ f'(x) &= 2x + e^x \end{aligned}$$

b)

$$\begin{aligned} g(x) &= x^3 \cdot \sqrt{2x-1} \\ g'(x) &= 3x^2 \cdot \sqrt{2x-1} + x^3 \cdot \frac{1}{2} \cdot (2x-1)^{-\frac{1}{2}} \cdot 2 \\ &= x^2\left(3\sqrt{2x-1} + \frac{x}{\sqrt{2x-1}}\right) \\ &= x^2\left(\frac{3\sqrt{2x-1}\sqrt{2x-1}}{\sqrt{2x-1}} + \frac{x}{\sqrt{2x-1}}\right) \\ &= x^2\left(\frac{3(2x-1)+x}{\sqrt{2x-1}}\right) \\ &= \frac{x^2(7x-3)}{\sqrt{2x-1}} \end{aligned}$$

c)

$$\begin{aligned} h(x) &= \frac{x^3}{x^2-1} \\ h'(x) &= \frac{3x^2 \cdot (x^2-1) - x^3 \cdot 2x}{(x^2-1)^2} \\ &= \frac{x^2(3x^2 - 3 - 2x^2)}{(x^2-1)^2} \\ &= \frac{x^2(x^2-3)}{(x^2-1)^2} \end{aligned}$$

Oppgave 2 (2 poeng)

$$\begin{aligned} 2\ln(a^3b^4) - 3\ln a - 3\ln(ab^2) &= 2(3\ln a + 4\ln b) - 3\ln a - 3(\ln a + 2\ln b) \\ &= 6\ln a + 8\ln b - 3\ln a - 3\ln a - 6\ln b \\ &= 2\ln b \end{aligned}$$

Oppgave 3 (5 poeng)

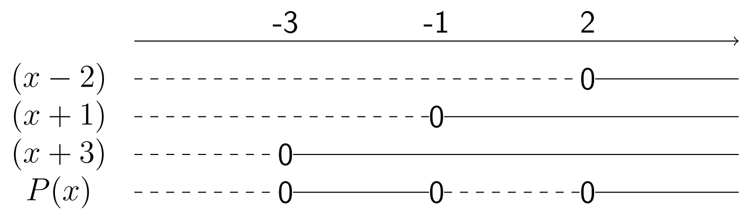
a)

$$x^3 + 2x^2 = 5x + 6$$
$$x^3 + 2x^2 - 5x - 6 = 0$$

$$x = 2:$$
$$2^3 + 2 \cdot 2^2 - 5 \cdot 2 - 6 = 8 + 8 - 10 - 6 = 0$$

b) $(x^3 + 2x^2 - 5x - 6) : (x - 2) = x^2 + 4x + 3 = (x + 1)(x + 3)$

$$x^3 + 2x^2 < 5x + 6$$
$$x^3 + 2x^2 - 5x - 6 < 0$$
$$(x - 2)(x + 1)(x + 3) < 0$$



$x \in \langle \leftarrow, -3 \rangle \cup \langle -1, 2 \rangle$

c)
$x \in \langle \leftarrow, -1 \rangle \cup [2, \rightarrow \rangle$

$$\frac{x - 2}{x + 1} \geq 0$$

Oppgave 4 (7 poeng)

a)

$$\vec{r}(t) = [4t + 2, 2t - 5t^2]$$
$$\vec{v}(t) = \vec{r}\,'(t) = [4, 2 - 10t]$$
$$\vec{a}(t) = \vec{v}\,'(t) = [0, -10]$$

b)

$$\vec{v} \perp \vec{a}$$
$$[4, 2 - 10t] \cdot [0, -10] = 0$$
$$(2 - 10t)(-10) = 0$$
$$2 - 10t = 0$$
$$10t = 2$$
$$t = \frac{1}{5}$$

Oppgave 5 (5 poeng)

A(1,1) B(6,2) C(5,7) D(2,8)

a)

$$\begin{aligned}\overrightarrow{AB} &= [6-1, 2-1] = [5,1] \\ |\overrightarrow{AB}| &= |[5,1]| = \sqrt{5^2+1^2} = \sqrt{26}\end{aligned}$$

b)

$$\begin{aligned}\overrightarrow{OT} &= \frac{1}{4}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}) \\ &= \frac{1}{4}([1,1]+[6,2]+[5,7]+[2,8]) \\ &= \frac{1}{4}[1+6+5+2, 1+2+7+8] \\ &= \frac{1}{4}[14,18] \\ &= \left[\frac{14}{4}, \frac{18}{4}\right] \\ &= \left[\frac{7}{2}, \frac{9}{2}\right] \\ T &= \left(\frac{7}{2}, \frac{9}{2}\right)\end{aligned}$$

c)
E(0,3) F(2,-2) G(7,3) H(x,y) T(3,2)

$$\begin{aligned}\overrightarrow{OT} &= \frac{1}{4}(\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}) \\ [3,2] &= \frac{1}{4}([0,3]+[2,-2]+[7,3]+[x,y]) \\ 4[3,2] &= [0+2+7+x, 3-2+3+y] \\ [12,8] &= [x+9, y+4] \\ x+9 = 12 &\wedge y+4=8 \\ x = 3 &\wedge y = 4 \\ H &= (3,4)\end{aligned}$$

Oppgave 6 (4 poeng)

Mulige utfall = RR,RB,BR,BB

$P(RR) = \frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6}$

$P(RB) = \frac{2}{4} \cdot \frac{2}{3} = \frac{1}{3}$

$P(BR) = \frac{2}{4} \cdot \frac{2}{3} = \frac{1}{3}$

$P(BB) = \frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6}$

$P(\text{2 like}) = \frac{1}{3}$

$P(\text{2 ulike}) = \frac{2}{3}$

$P(\text{2 ulike med 3 røde}) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{5} > \frac{1}{2}$

$P(\text{2 ulike med 4 røde}) = 2 \cdot \frac{4}{6} \cdot \frac{2}{5} = \frac{16}{30} = \frac{8}{15} > \frac{1}{2}$

$P(\text{2 ulike med 5 røde}) = 2 \cdot \frac{5}{7} \cdot \frac{2}{6} = \frac{20}{42} < \frac{1}{2}$

Oppgave 7 (6 poeng)

$$
\begin{aligned}
f(x) &= (x+2)(x^2+x+5) \\
&= x^3+2x^2+x^2+2x+5x+10 \\
&= x^3+3x^2+7x+10
\end{aligned}
$$

a)

$$
\begin{aligned}
f'(x) &= 3x^2+6x+7 \\
f'(x) &= 0 \\
3x^2+6x+7 &= 0 \\
x &= \frac{-6 \pm \sqrt{6^2-4\cdot 3\cdot 7}}{2\cdot 3} \\
&= \frac{-6 \pm \sqrt{36-64}}{6}
\end{aligned}
$$

ingen løsning, altså er grafen enten alltid positiv (stigende) eller alltid negativ (synkende).

Vi vet at dette er en 3.gradsfunksjon med positivt x^3-ledd, altså vil den være stigende for alle verdier av x.

b)

$$
\begin{aligned}
f''(x) &= 6x+6 \\
f''(x) &= 0 \\
6(x+1) &= 0 \\
x &= -1
\end{aligned}
$$

Grafen til f(x) har et vendepunkt når x=-1

c)
Siden grafen er stigende for alle verdier av x, så har den kun ett nullpunkt, $f(x)=0 \Rightarrow x=-2$. Vi vet også at den krysser y-skasen i $y=10$, da dette er konstantleddet til $f(x)$.

Oppgave 8 (2 poeng)

Vi får opplyst i oppgaven at $\angle BSC = 140^o$, siden $\angle BAC$ er perifervinkelen til denne får vi at $\angle BAC = 70^o$.

$\triangle BCS$ er likebeint fordi 2 av sidene er radien i sirkelen. Da vet vi at $\angle SBC = \frac{1}{2}(180^o - 140^o) = 20^o$.

Da vet vi at $\angle ABC = 30^o + 20^o = 50^o$, og at $\angle BCA = 180^o - 50^o - 70^o = 60^o$.

Oppgave 9 (2 poeng)

Areal av et trapes :

$$F = \frac{a+b}{2} \cdot h$$

$$\begin{aligned}
F1 &= \frac{AB+CD}{2} \cdot h \\
F2 &= \frac{AB+MN}{2} \cdot \frac{h}{2} + \frac{MN+CD}{2} \cdot \frac{h}{2} \\
&= \frac{AB+MN}{4} \cdot h + \frac{MN+CD}{4} \cdot h
\end{aligned}$$

$$\begin{aligned}
F1 &= F2 \\
\frac{AB+CD}{2} \cdot h &= \frac{AB+MN}{4} \cdot h + \frac{MN+CD}{4} \cdot h \\
2(AB+CD) &= AB+MN+MN+CD \\
2AB+2CD &= 2MN+AB+CD \\
2MN &= 2AB+2CD-AB-CD \\
MN &= \frac{AB+CD}{2}
\end{aligned}$$