Eksamen R1
Vår 2020
Løsningsforslag

DEL 1

Oppgave 1 (5 poeng)

a) $f(x) = x^6 + 3x^5 + \ln(x)$

$f'(x) = 6x^5 + 15x^4 + \frac{1}{x}$

b) $g(x) = 2x^2 \cdot e^{2x-1}$

$g'(x) = 4x \cdot e^{2x-1} + 2x^2 \cdot e^{2x-1} \cdot 2 = 4xe^{2x-1}(1+x)$

c) $h(x) = \frac{4x-1}{x+2}$

$h'(x) = \frac{4(x+2)-(4x-1)\cdot 1}{(x+2)^2} = \frac{4x+8-4x+1}{(x+2)^2} = \frac{9}{(x+2)^2}$

Oppgave 2 (4 poeng)

a)

$$\begin{aligned} \ln(x^2) + \ln(x) &= 12 \\ 2\ln x + \ln x &= 12 \\ 3\ln x &= 12 \\ \ln x &= 4 \\ x &= e^4 \end{aligned}$$

b)

$$\begin{aligned} e^{2x} - e^x &= 6 \\ (e^x)^2 - e^x - 6 &= 0 \\ (e^x - 3)(e^x + 2) &= 0 \\ e^x &= 3 \vee e^x = -2 \\ x &= \ln(3) \vee \text{ikke gyldig - kan ikke ta logaritme til negativt tall} \end{aligned}$$

Oppgave 3 (4 poeng)

$\vec{u} \cdot \vec{v} = -2$, $|\vec{u}| = 3, |\vec{v}| = 2$
$\vec{a} = 2\vec{u} + 3\vec{v}$, $\vec{b} = t \cdot \vec{u} + 5\vec{v}$

a)

$$\begin{aligned}
\vec{a}||\vec{b} &\Rightarrow k\cdot\vec{a}=\vec{b}\\
k\cdot(2\vec{u}+3\vec{v}) &= t\cdot\vec{u}+5\vec{v}\\
k\cdot 2\vec{u} = t\cdot\vec{u} &\vee k\cdot 3\vec{v}=5\vec{v}\\
3k &= 5\\
k &= \frac{5}{3}\\
\frac{5}{3}\cdot 2 &= t\\
t &= \frac{10}{3}
\end{aligned}$$

b)

$$\begin{aligned}
\vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b} &= 0\\
(2\vec{u}+3\vec{v})\cdot(t\cdot\vec{u}+5\vec{v}) &= 0\\
2t\vec{u}^2+10\vec{u}\vec{v}+3t\vec{u}\vec{v}+15\vec{v}^2 &= 0\\
2t\cdot 3^2+10(-2)+3t(-2)+15\cdot 2^2 &= 0\\
19t-20-6t+60 &= 0\\
12t &= -14\\
t &= -\frac{10}{3}
\end{aligned}$$

Oppgave 4 (7 poeng)

$$P(x) = 6x^3 - 5x^2 - 2x + 1$$

a) Divisjonen $P(x) : (x-1)$ går opp dersom $(x-1)$ er en faktor i $P(x)$, og da vil $P(1) = 0$.

$$P(1) = 6 \cdot 1^3 - 5 \cdot 1^2 - 2 \cdot 1 + 1 = 6 - 5 - 2 + 1 = 0$$

b)

$$\begin{array}{l} \big(\ 6x^3 - 5x^2 - 2x + 1\big) : \big(x-1\big) = 6x^2 + x - 1 \\ \underline{-6x^3 + 6x^2} \\ \qquad\quad x^2 - 2x \\ \qquad \underline{-x^2 \ \ + x} \\ \qquad\qquad -x + 1 \\ \qquad\qquad \underline{\ \ \ x - 1} \\ \qquad\qquad\qquad 0 \end{array}$$

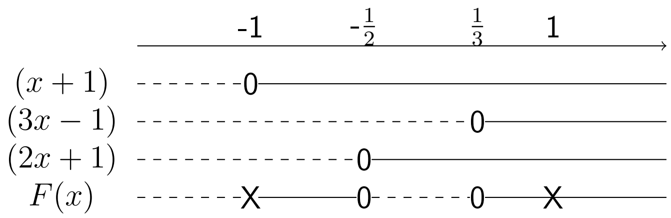
$$\begin{aligned} 6x^2 + x - 1 &= 0 \\ x &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 6 \cdot (-1)}}{2 \cdot 6} \\ x &= \frac{-1 \pm \sqrt{25}}{12} \\ x &= \frac{-1 \pm 5}{12} \\ x = \frac{4}{12} &\vee x = \frac{-6}{12} \\ x = \frac{1}{3} &\vee x = \frac{-1}{2} \\ 6x^2 + x - 1 &= 6(x - \frac{1}{3})(x + \frac{1}{2}) \\ &= (3x-1)(2x+1) \end{aligned}$$

Da får vi at :

$$P(x) = 6x^3 - 5x^2 - 2x + 1 = (x-1)(3x-1)(2x+1)$$

c)

$$\begin{aligned} F(x) &\leq 0 \\ \frac{P(x)}{x^2 - 1} &\leq 0 \\ \frac{(x-1)(3x-1)(2x+1)}{(x+1)(x-1)} &\leq 0 \\ \frac{(3x-1)(2x+1)}{(x+1)} &\leq 0 \end{aligned}$$



$F(x) \leq 0$ når $x \in \big\langle \leftarrow, -1 \big\rangle \cup \big[-\frac{1}{2}, \frac{1}{3} \big]$

$F(x) \geq 0$ når $x \in \big\langle -1, -\frac{1}{2} \big] \cup \big[\frac{1}{3}, 1 \big\rangle \cup \big\langle 1, \rightarrow \big\rangle$

d)

$$\begin{aligned}
\lim_{x \to 1} F(x) &= \lim_{x \to 1} \frac{(x-1)(3x-1)(2x+1)}{(x+1)(x-1)} \\
&= \lim_{x \to 1} \frac{(3x-1)(2x+1)}{(x+1)} \\
&= \frac{2 \cdot 3}{2} \\
&= 3
\end{aligned}$$

$$\begin{aligned}
\lim_{x \to -1} F(x) &= \lim_{x \to -1} \frac{(x-1)(3x-1)(2x+1)}{(x+1)(x-1)} \\
&= \lim_{x \to -1} \frac{(3x-1)(2x+1)}{(x+1)} \\
&= \frac{(-4)(-1)}{0} \\
&= \infty
\end{aligned}$$

altså ingen grenseverdi.

Oppgave 5 (5 poeng)

a) $\binom{8}{3} = \frac{8!}{(8-3)!\cdot 3!} = \frac{8\cdot 7\cdot \not{6}}{\not{3}\cdot\not{2}} = 56$

b) $\binom{8}{4} = \frac{8!}{(8-4)!\cdot 4!} = \frac{8\cdot 7\cdot \not{6}\cdot 5}{4\cdot\not{3}\cdot\not{2}} = 2\cdot 7\cdot 5 = 70$

$$\begin{aligned}
P(X=3) &= \frac{\binom{3}{3}\cdot\binom{5}{1}}{\binom{8}{4}} \\
&= \frac{1\cdot 5}{70} \\
&= \frac{1}{14}
\end{aligned}$$

c) $\binom{3}{2} = \frac{3!}{(3-2)!\cdot 2!} = \frac{3\cdot 2}{2} = 3$

$\binom{5}{2} = \frac{5!}{(5-2)!\cdot 2!} = \frac{5\cdot 4}{2} = 10$

$$\begin{aligned}
P(X=2) &= \frac{\binom{3}{2}\cdot\binom{5}{2}}{\binom{8}{4}} \\
&= \frac{3\cdot 10}{70} \\
&= \frac{3}{7} \\
P(X\geq 2) &= P(X=2)+P(X=3) \\
&= \frac{3}{7}+\frac{1}{14} \\
&= \frac{6}{14}+\frac{1}{14} \\
&= \frac{7}{14} \\
&= \frac{1}{2}
\end{aligned}$$

Oppgave 6 (4 poeng)

$$f(x) = 9 - x^2$$

a) Areal trapes $= \frac{7}{a+b}2 \cdot h$

$$\begin{aligned} F(x) &= \frac{6+2x}{2} \cdot f(x) \\ &= (3+x)(9-x^2) \\ &= 27 - 3x^2 + 9x - x^3 \\ &= -x^3 - 3x^2 + 9x + 27 \quad \text{, som skulle vises} \end{aligned}$$

b)

$$\begin{aligned} F'(x) &= 0 \\ -3x^2 - 6x + 9 &= 0 \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \\ x = -3 &\vee x = 1 \\ x &= -3 \text{ er utenfor definisjonsområdet} \\ F(1) &= -1 - 3 + 9 + 27 = 32 \end{aligned}$$

(tegn fortegnslinje for å se topp/bunn)

Max areal er altså $F(1) = 8$

Oppgave 7 (3 poeng)

$\angle BDA = \angle u =$ perifervinkel til $\angle w$.

Da blir $\angle u = 65°$ og $\angle w = 130°$

$\angle CEB + u + 35° = 180°$

$\angle CEB = 180° - 35° - 65° = 80°$

$v = 180° - \angle CEB = 180° - 80° = 100°$

Oppgave 8 (4 poeng)

a)
$A(-1, 1)$, $C = (7, 5)$
$\overrightarrow{AC} = [7 + 1, 5 - 1] = [8, 4]$
$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

Linja går gjennom punktet $(0, 1)$ og har retningsvektoren $\vec{r} = [1, 2]$

$$l : \begin{matrix} x = t \\ y = 1 + 2t \end{matrix}$$

Da blir $D(t, 1 + 2t)$

$\overrightarrow{AD} = [t - (-1), 1 + 2t - 1] = [t + 1, 2t]$
b)

$$\begin{aligned}
\overrightarrow{CD} &= [t - 7, 2t - 4] \\
|\overrightarrow{CD}| &= |\overrightarrow{AD}| \\
\sqrt{(t - 7)^2 + (2t - 4)^2} &= \sqrt{(t + 1)^2 + (2t)^2} \\
t^2 - 14t + 49 + 4t^2 - 16t + 16 &= t^2 + 2t + 1 + 4t^2 \\
32t &= 64 \\
t &= 2 \\
D &= (2, 5) \\
\overrightarrow{CD} &= [2 - 7, 4 - 4] = [-5, 0] \\
\overrightarrow{AB} &= [5, 0] = [x_B - (-1), y_B - 1] \\
x_B &= 5 - 1 = 4 \\
y_B &= 0 + 1 = 1 \\
B &= (4, 1)
\end{aligned}$$

DEL 2

Oppgave 1 (6p.)

a) Binomisk forsøk forutsetter lik sannsynlighet for albil hele tiden. Ma da anta at disse 100 bilene representerer bilmassen, slik at vi kan anse dette som et forsøk MED tilbakelegging.

b) $P(X = 15) = \binom{100}{15} \cdot 0,125^{15} \cdot (1 - 0,125)^8 5$

$P(X \geq 15)$

c)
Vi prøver oss fram med sannsynlighetskalkulatoren $P(X \geq 20) > 0,9 \Rightarrow n = 204$

Oppgave 2 (4 poeng)

a)

$$\begin{aligned} p(x) &:= x^2 + 3x - 1 \\ A &:= (-1, p(-1)) = (-1, -3) \\ B &:= (3, p(3)) = 3, 17 \\ Linje[A, B] &\rightarrow y = 5x + 2 \\ Tangent((1, 3), p) &\rightarrow y = 5x - 2 \\ \text{samme stigningstall altså parallelle} \end{aligned}$$

b)
$q(x) := a \cdot x^2 + b \cdot x + c$

$q'(x) = \frac{q(x+h) - q(x-h)}{2h}$

Oppgave 3 (8 poeng)

$\vec{r_1} = [t^2 - 2, t^3 - 2t]$, $t \in [-2, 2]$

$\vec{r_2} = [2t - 1, 4t - 4t^2]$, $t \in [-2, 2]$

a)
$r1(t) := Kurve[t^2 - 2, t^3 - 2t, t, -2, 2]$

b) $v1(t) := Derivert(r1(t))$
$r1(s) := Kurve[2s - 1, 4s - 4s^2, s, -2, 2]$
$v2(s) := Derivert(r2(s))$
$abs(v1(-1)) = \sqrt{5}$
$abs(v2(-1)) = 2\sqrt{37}$

c)
$v1(t) || v2(s) \Rightarrow v1(t) = k \cdot v2(s)$

$Løs((2,-8t+4)=k\cdot(2t,3t^2-2))$

$t=0,65 \;\vee\; t=-0,28$

d)
Avstand mellom partiklene :

$P1=(t^2-2,t^3-2t)$

$P2=(2t-1,4t,4t^2)$