Eksamen R1
Vår 2021
Løsningsforslag

DEL 1

Oppgave 1 (6 poeng)

a)

$$
\begin{aligned}
f(x) &= 3x^3 - 4x + \frac{1}{x} \\
&= 3x^3 - 4x + x^{-1} \\
f'(x) &= 9x^2 - 4 - x^{-2} \\
&= \underline{\underline{9x^2 - 4 - \frac{1}{x^2}}}
\end{aligned}
$$

b)

$$
\begin{aligned}
f(x) &= 3x^2 \cdot \ln x \\
f'(x) &= 6x \cdot \ln x + 3x^2 \cdot \frac{1}{x} \\
&= 6x \cdot \ln x + 3x \\
&= \underline{\underline{3x(2\ln x + 1)}}
\end{aligned}
$$

c)

$$
\begin{aligned}
f(x) &= \sqrt{4x^2 - 5} \\
&= (4x^2 - 5)^{\frac{1}{2}} \\
f'(x) &= \frac{1}{2}(4x^2 - 5)^{-\frac{1}{2}} \cdot (8x) \\
&= \underline{\underline{\frac{8x}{2\sqrt{4x^2 - 5}}}}
\end{aligned}
$$

Oppgave 2 (4 poeng)

a)

$$\begin{aligned}
\frac{3x}{x^2-x-2}-\frac{2x}{x+1}-\frac{2}{x-2} &= \frac{3x}{(x-2)(x+1)}-\frac{2x}{x+1}-\frac{2}{x-2}\\
&= \frac{3x-2x(x-2)-2(x+1)}{(x-2)(x+1)}\\
&= \frac{3x-2x^2+4x-2x-2}{(x-2)(x+1)}\\
&= \frac{-2x^2+5x-2}{(x-2)(x+1)}
\end{aligned}$$

$$\begin{aligned}
\text{MR}:\ -2x^2+5x-2 &= 0\\
x &= \frac{-5\pm\sqrt{5^2-4\cdot(-2)\cdot(-2)}}{2\cdot(-2)}\\
&= \frac{-5\pm\sqrt{25-16}}{-4}\\
&= \frac{-5\pm\sqrt{9}}{-4}\\
&= \frac{-5\pm 3}{-4}\\
x = \frac{1}{2} &\vee x = 2\\
-2x^2+5x-2 = -2(x-\frac{1}{2})(x-2) &= -(2x-1)(x-2)
\end{aligned}$$

....

$$\begin{aligned}
&= \frac{-(2x-1)(x-2)}{(x-2)(x+1)}\\
&= \underline{\underline{\frac{-2x+1}{x+1}}}
\end{aligned}$$

b)

$$\begin{aligned}
\ln(a\cdot b^2)-8\ln b+2\ln a^3-3\ln\frac{a^2}{b^2} &= \ln a+2\ln b-8\ln b+6\ln a-3(2\ln a-2\ln b)\\
&= \ln a+2\ln b-8\ln b+6\ln a-6\ln a+6\ln b\\
&\underline{\underline{= \ln a}}
\end{aligned}$$

Oppgave 3 (4 poeng)

a)

$$\begin{aligned}
\vec{a}\cdot\vec{b} &= [4,1]\cdot[-1,3]\\
&= -4+3\neq 0
\end{aligned}$$

, altså ikke orthogonale

b)

$$\begin{aligned}
\vec{c} &= r \cdot \vec{a} + s \cdot \vec{b} \\
[4, 14] &= r \cdot [4, 1] + s \cdot [-1, 3] \\
[4, 14] &= [4r - s, r + 3s] \\
4 = 4r - s \wedge 14 &= r + 3s \\
s &= 4r - 4 \\
r + 3(4r - 4) &= 14 \\
r + 12r - 12 &= 14 \\
13r &= 26 \\
r &= 2 \\
s &= 4(2 - 1) = 4
\end{aligned}$$

Oppgave 4 (4 poeng)

A = maskin A
B = feil
$P(A) = 0,4$
$P(\bar{A}) = 0,6$
$P(B|A) = 0,2$
$P(B|\bar{A}) = 0,1$
$P(B) = P(B \cap A) + P(B \cap \bar{A})$

a) Sannsynligheten for feil på et tilfeldig deksel er :

$$\begin{aligned}
P(B) &= P(A \cap B) + P(\bar{A} \cap B) \\
&= P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A}) \\
&= 0,4 \cdot 0,2 + 0,6 \cdot 0,1 \\
&= 0,08 + 0,06 \\
&\underline{\underline{= 0,14}}
\end{aligned}$$

b) Gitt at dekselet har en feil, sannsynligheten for maskin A :

$$\begin{aligned}
P(A) \cdot P(B|A) &= 0,4 \cdot 0,2 = 0,08 \\
P(B) \cdot P(A|B) &= 0,08 \\
P(A|B) &= \frac{P(A) \cdot P(B|A)}{P(B)} \\
&= \frac{0,08}{0,14} \\
&\underline{\underline{= \frac{4}{7}}}
\end{aligned}$$

Oppgave 5 (6 poeng)

a)

$$
\begin{aligned}
f(x) &= x^3 - 14x + 15 \\
f'(x) &= 3x^2 - 14 \\
f'(x) &= 0 \\
3x^2 - 14 &= 0 \\
x^2 &= \frac{14}{3} \\
x &= \pm\sqrt{\frac{14}{3}} \\
x &= \pm\frac{\sqrt{14}\sqrt{3}}{3} \\
x &= \pm\frac{\sqrt{42}}{3}
\end{aligned}
$$

Grafen til f har toppunkt når $x = -\frac{\sqrt{42}}{3}$ og bunnpunkt for $x = \frac{\sqrt{42}}{3}$

b)

Ser av grafen at grafen har et nullpunkt i $x = 3$, sjekker ved regning : $f(3) = 3^3 - 14 \cdot 3 + 15 = 27 - 42 + 15 = 0$

Utfører polynomdivisjon :

$$(x^3 - 14x + 15) : (x - 3) = x^2 + 3x - 5$$

De andre nullpunktene blir da :

$$
\begin{aligned}
x^2 + 3x - 5 &= 0 \\
x &= \frac{-3 \pm \sqrt{3^2 - 4 \cdot (-5)}}{2} \\
&= \frac{-3 \pm \sqrt{29}}{2} \\
f(x) &> 0 \\
x &\in \underline{\underline{\left\langle \frac{-3 - \sqrt{29}}{2}, \frac{-3 + \sqrt{29}}{2} \right\rangle \cup \langle 3, \rightarrow \rangle}}
\end{aligned}
$$

Oppgave 6 (3 poeng)

$$
\begin{aligned}
f(x) &= \frac{8}{x^2+4} \\
\text{Areal av rektangelet :} \\
A(t) &= t \cdot f(t) \\
&= \frac{8t}{t^2+4} \\
A'(t) &= \frac{8(t^2+4) - 8t \cdot 2t}{(t^2+4)^2} \\
&= \frac{8t^2+32-16t^2}{(t^2+4)^2} \\
&= \frac{32-8t^2}{(t^2+4)^2} \\
&= \frac{8(4-t^2)}{(t^2+4)^2} \\
A'(t) &= 0 \\
\frac{8(4-t^2)}{(t^2+4)^2} &= 0 \\
4-t^2 &= 0 \\
t &= \pm 2
\end{aligned}
$$

Altså vil arealet være størst når $\underline{\underline{t=2}}$, $(t>0)$

Oppgave 8 (3 poeng)

a)
$\angle BED$ er perifervinkel til buen BD, altså er $\angle BED = \frac{108^\circ}{2} \underline{\underline{= 54^\circ}}$

b)
$\angle BED = 54^\circ$
$\angle EBF = 60^\circ$, fordi $\triangle ABC$ er likesidet trekant.
$\angle CFD = \angle BFE = 180^\circ - \angle BED - \angle EBF = 180^\circ - 54^\circ - 60^\circ \underline{\underline{= 66^\circ}}$

Oppgave 9 (5 poeng)

a)

$\overrightarrow{AB} = [4-(-2), 3-1] = [6,2] = 2[3,1]$

$$
l : \begin{cases} x &= -2+3t \\ y &= 1+t \end{cases}
$$

b)
Skjæring med x-aksen når $y = 0 : 1 + t = 0 \Rightarrow t = -1$ altså i $\underline{\underline{(-5, 0)}}$

Skjæring med y-aksen når $x = 0 : -2 + 3t = 0 \Rightarrow t = \frac{2}{3}$ altså i $\underline{\underline{(0, \frac{5}{3})}}$

c)
Radien i sirkelen er avstand fra S til l :

$$
\begin{aligned}
\overrightarrow{AB} &= [6, 2] = 2[3, 1] \\
\overrightarrow{PS} &= [1 - (-2 + 3t), 0 - (1 + t)] \\
&= [3 - 3t, -1 - t] \\
\overrightarrow{AB} &\perp \overrightarrow{PS} \\
[3, 1] \cdot [3 - 3t, -1 - t] &= 0 \\
3(3 - 3t) + (-1 - t) &= 0 \\
9 - 9t - 1 - t &= 0 \\
10t &= 8 \\
t &= \frac{4}{5} \qquad = \\
\overrightarrow{PS} &= [3 - 3 \cdot \frac{4}{5}, -1 - \frac{4}{5}] \\
&= [\frac{15 - 12}{5}, \frac{-5 - 4}{5}] \\
&= [\frac{3}{5}, -\frac{9}{5}] \\
r = |\overrightarrow{PS}| &= \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{9}{5}\right)^2} \\
&= \sqrt{\frac{9 + 81}{25}} \\
&= \sqrt{\frac{90}{25}} \\
&= \frac{3\sqrt{10}}{5}
\end{aligned}
$$

Altså er radien til sirkelen : $\underline{\underline{r = \frac{3\sqrt{10}}{5}}}$

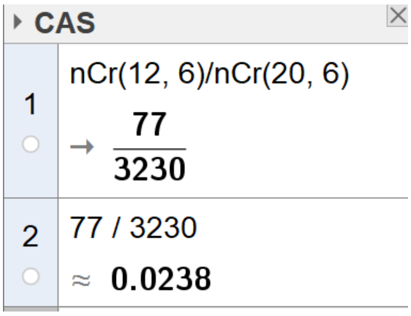
DEL2

Oppgave 1 (6 poeng)

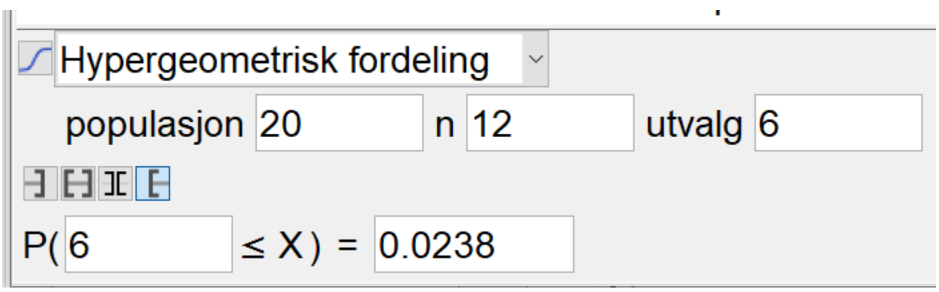
20 ministre inkl. statsministeren.
12 H, 4 V, 4 KrF.
6 skal trekkes ut.
X=antall fra høyre.

a)
Uten tilbakelegging - Hypergeometrisk

$P(X=6)=\frac{\binom{12}{6}\cdot\binom{8}{0}}{\binom{20}{6}}$



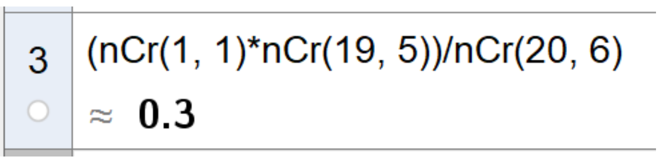
eller vi kan bruke sannsynlighetskalkulatoren :



Sannsynligheten for at alle 6 er fra høyre er : $\underline{\underline{P(X \geq 6) = 0,238}}$

b)
Y=1 betyr at statsmonosteren er med.

$P(Y=1)=\frac{\binom{1}{1}\cdot\binom{19}{5}}{\binom{20}{6}}$



c)

Sannsynligheten for at det blir 2 fra hvert parti :

$$\frac{\binom{12}{2}\cdot\binom{4}{2}\cdot\binom{4}{2}}{\binom{20}{6}} = \underline{\underline{0,0613}}$$

4 (nCr(12, 2)*nCr(4, 2)*nCr(4, 2))/(nCr(20, 6))
$\approx$ **0.0613**

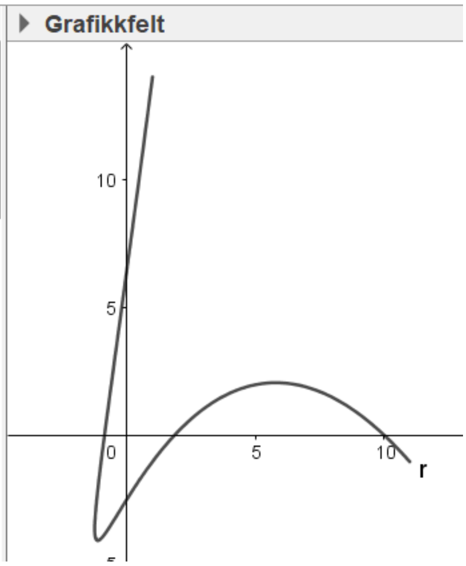
Oppgave 2 (6 poeng)

a)
Banen til partiklen :

CAS

1 r(t):=Kurve(t^2-7t+11, t^3-6t^2+8t-1, t, 0, 5)
$\rightarrow$ $\mathbf{r := (t^2 - 7\,t + 11, t^3 - 6\,t^2 + 8\,t - 1)}$

2



b)

CAS

1 r(t):=Kurve(t^2-7t+11, t^3-6t^2+8t-1, t, 0, 5)
$\rightarrow$ $\mathbf{r := (t^2 - 7\,t + 11, t^3 - 6\,t^2 + 8\,t - 1)}$

2 v(t):=Derivert(r(t))
$\rightarrow$ $\mathbf{v(t) := \left(2\,t - 7, 3\,t^2 - 12\,t + 8\right)}$

3 abs(v(1))
$\rightarrow$ $\mathbf{\sqrt{26}}$

c)

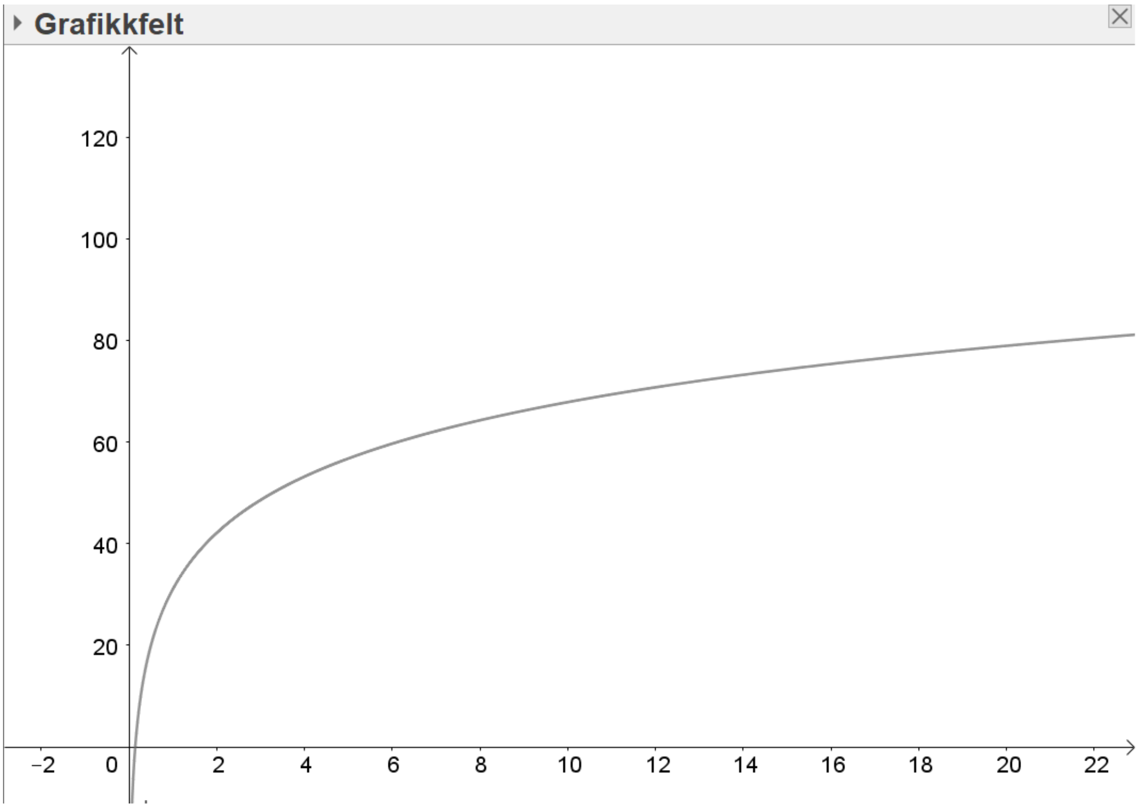
2	v(t):=Derivert(r(t)) → $v(t) := (2t - 7, 3t^2 - 12t + 8)$
3	abs(v(1)) → $\sqrt{26}$
4	Ekstremalpunkt(v, -10, 0) → $(-3, -4)$

Oppgave 3 (6 poeng)

a)

CAS	
1	h(a):=16 ln(a)+31 → $h(a) := 16\ \ln(a) + 31$
2	Løs(h=65) → $\left\{a = \sqrt[8]{e^{17}}\right\}$
3	{a = nrot(e^{17},8)} ≈ $\{a = 8.37\}$

b)



c)

4	L(y):=16ln(y) + 31 → $L(y) := 16 \ln(y) + 31$
5	F(x):=16ln(x) + 31 → $F(x) := 16 \ln(x) + 31$
6	L(y - 1) = F(x - 1) + 20 → $16 \ln(y-1) + 31 = 16 \ln(x-1) + 51$
7	L(y)=F(x)+10 → $16 \ln(y) + 31 = 16 \ln(x) + 41$
8	NLøs({$6, $7}) → $\{x = 1.54, y = 2.87\}$
9	L(2.87) ≈ 47.87

Oppgave 4 (6 poeng)

a)

CAS	
1	a:=5 → $a := 5$
2	b:=7 → $b := 7$
3	c:=8 → $c := 8$
4	s:=(a+b+c)/2 → $s := 10$
5	F=sqrt(s(s-a)(s-b)(s-c)) → $F = 10\sqrt{3}$
6	F = 10sqrt(3) ≈ $F = 17.32$

b)

$$\begin{aligned}
q &= c - p \\
q^2 &= (c-p)^2 \\
&= c^2 - 2cp + p^2 \\
b^2 &= h^2 + q^2 \text{ - Pythagoras} \\
&= h^2 + c^2 - 2cp + p^2 \text{ - som skulle vises.}
\end{aligned}$$

c)

$$a^2 = h^2 + p^2 \Rightarrow h^2 = p^2 - a^2$$
$$b^2 = h^2 + c^2 - 2cp + p^2 \Rightarrow h^2 = b^2 - c^2 + 2cp - p^2$$
$$a^2 - p^2 = b^2 - c^2 + 2cp - p^2$$
$$a^2 = b^2 - c^2 + 2cp$$
$$2cp = a^2 - b^2 + c^2$$
$$p = \frac{a^2 - b^2 + c^2}{2c} \text{ , som skulle vises}$$

d) Areal av trekant :

$$
\begin{aligned}
F &= \frac{1}{2}\cdot c\cdot h \\
CAS &: \\
p &:= (a^2-b^2+c^2)/(2c) \\
h &:= \sqrt{p^2-a*2} \\
F &:= 1/2\cdot c\cdot h \\
\to F &= \frac{1}{4}c\sqrt{4a^2-\left(\frac{2x^2-2b^2+2c^2}{2c}\right)^2} \\
F &:= \sqrt{s(s-a)(s-b)(s-c)}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{1}{4}c\sqrt{4a^2-\left(\frac{2a^2-2b^2+2c^2}{2c}\right)^2}\right)^2 &= (\sqrt{s(s-a)(s-b)(s-c)})^2 \\
\frac{1}{4}\cdot\frac{1}{4}\cdot c^2\left(4a^2-\left(\frac{a^2-b^2+c^2}{c}\right)^2\right)^2 &= s(s-a)(s-b)(s-c) \\
\frac{1}{4}\cdot c^2\left(4a^2\cdot\frac{1}{4}-\frac{1}{4}\cdot\left(\frac{a^2-b^2+c^2}{c}\right)^2\right)^2 &= s(s-a)(s-b)(s-c) \\
\frac{1}{4}\cdot c^2\left(a^2-\left(\frac{a^2-b^2+c^2}{2c}\right)^2\right)^2 &= s(s-a)(s-b)(s-c)
\end{aligned}
$$