



Arbeidshefte

R1 Eksamensoppgaver

Algebra og derivasjon

 = uten hjelpemidler

 = med hjelpemidler

Oppgave 1

Skriv så enkelt som mulig

a) $\frac{x^2+x}{x^2-4} - \frac{2}{4-2x} =$

b) $\frac{2^{-4} \cdot 2^3}{2^{-2}} =$

c) $\frac{x-2}{x^2+2x} - \frac{x+2}{x^2-2x} - \frac{4x}{x^2-4} =$

d) $\frac{2x+10}{x^2-25} + \frac{x}{x+5} - \frac{2}{x-5} =$

Oppgave 2

Skriv så enkelt som mulig

a) $\lg(x \cdot y^2) - 2 \lg y + \lg\left(\frac{x}{y^2}\right) =$

b) $\lg\left(\frac{1}{a^2}\right) + 3 \lg a =$

c) $\lg(a^2b) - \lg\left(\frac{1}{ab}\right) =$

d) $\lg(a^2b) + \lg(ab^2) + \lg\left(\frac{a}{b^3}\right) =$

e) $\ln(a^2 \cdot b) - 2 \ln a - \ln\left(\frac{1}{b}\right) =$

Oppgave 3

Deriver funksjonene

a) $f(x) = x^2 \cdot e^{2x}$

b) $f(x) = x \cdot \ln x$

c) $f(x) = 3e^{x^2+1}$

d) $f(x) = x^2 \cdot \ln x$

e) $f(x) = \frac{2x-1}{x+1}$

Oppgave 4

Deriver funksjonene

a) $f(x) = x \cdot e^{-x}$

b) $f(x) = 3 \cdot e^{2x}$

c) $f(x) = (x^2 + 1)^4$

d) $f(x) = x \cdot e^{2x}$

e) $f(x) = 5e^{3x}$

Oppgave 5

Deriver funksjonene

a) $f(x) = x^3 \cdot \ln(2x)$

b) $f(x) = 4e^{x^2-3x}$

c) $f(x) = 2x \cdot e^x$

d) $f(x) = 3\sqrt{x^2 - 1}$

e) $f(x) = \frac{500}{x} + 8x^2$

Oppgave 6

Deriver funksjonene

a) $f(x) = 3 \ln(2x)$

b) $f(x) = 3x \cdot e^{x^2}$

c) $f(x) = 0,02x^3 + 0,6x^2 + 4,1$

d) $f(x) = \sqrt{x^2 - 1}$

Oppgave 7

Deriver funksjonene

a) $f(x) = -\frac{1}{3}x^3 + 2x^2$

b) $f(x) = (x^2 + 3)^4$

c) $f(x) = x^4 - 4x^3$

d) $f(x) = x^2 \cdot e^{2x}$

Oppgave 8

Deriver uttrykkene

a) $A(r) = \pi r^2$

b) $V(r) = \frac{4}{3}\pi r^3$

c) $h(a) = \frac{2a^2}{e^a}$

Oppgave 9

Funksjonene f er gitt som $f(x) = x^2 + 1$

Bruk definisjonen av den deriverte til å vise at $f'(x) = 2x$.

Oppgave 10

Bestem grenseverdien, dersom den eksisterer

a) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1} =$

b) $\lim_{x \rightarrow 8} \frac{x^2 - 64}{2x - 16} =$

c) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x - 2} =$

Oppgave 11

Bestem grenseverdien, dersom den eksisterer

a) $\lim_{x \rightarrow -1} \frac{x^3 - x}{x + 1} =$

b) $\lim_{x \rightarrow 2} \frac{x^2 + 4}{x - 2} =$

c) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$

Oppgave 12

Vis at $h(x) = x^x$ kan skrives som $h(x) = e^{x \cdot \ln x}$

Oppgave 13

Løs likningene

a) $2 \ln x - 4 = 0$

b) $e^{2x} - 3e^x + 2 = 0$

c) $4 \cdot \left(1 + \frac{x}{100}\right)^x = 64$

d) $3^{4x} + 7 = 34$

e) $\lg x + \lg(x - 1) = \lg 2$

FASIT

Oppgave 1

Skriv så enkelt som mulig

a)

$$\begin{aligned}
 \frac{x^2 + x}{x^2 - 4} - \frac{2}{4 - 2x} &= \frac{x^2 + x}{(x + 2)(x - 2)} + \frac{2}{2(x - 2)} \\
 &= \frac{(x^2 + x) \cdot 2 + 2 \cdot (x + 2)}{2(x + 2)(x - 2)} \\
 &= \frac{2x^2 + 2x + 2x + 4}{2(x + 2)(x - 2)} \\
 &= \frac{2x^2 + 2x + 2x + 4}{2(x + 2)(x - 2)} \\
 &= \frac{2(x^2 + 2x + 2)}{2(x + 2)(x - 2)} \\
 &= \frac{(x^2 + 2x + 2)}{(x + 2)(x - 2)}
 \end{aligned}$$

b)

$$\frac{2^{-4} \cdot 2^3}{2^{-2}} = 2^{-4+3-(-2)} = 2$$

c)

$$\begin{aligned}
 \frac{x - 2}{x^2 + 2x} - \frac{x + 2}{x^2 - 2x} - \frac{4x}{x^2 - 4} &= \frac{x - 2}{x(x + 2)} - \frac{x + 2}{x(x - 2)} - \frac{4x}{(x + 2)(x - 2)} \\
 &= \frac{(x - 2)(x - 2) - (x + 2)(x + 2) - 4x^2}{x(x + 2)(x - 2)} \\
 &= \frac{x^2 - 4x + 4 - x^2 - 4x - 4 - 4x^2}{x(x + 2)(x - 2)} \\
 &= \frac{-4x^2 - 8x}{x(x + 2)(x - 2)} \\
 &= \frac{-4x(x + 2)}{x(x + 2)(x - 2)} \\
 &= \frac{-4}{x - 2}
 \end{aligned}$$

d)

$$\begin{aligned}
\frac{2x+10}{x^2-25} + \frac{x}{x+5} - \frac{2}{x-5} &= \frac{2x+10+x(x-5)-2(x+5)}{(x+5)(x-5)} \\
&= \frac{2x+10+x^2-5x-2x-10}{(x+5)(x-5)} \\
&= \frac{x^2-5x}{(x+5)(x-5)} \\
&= \frac{x(x-5)}{(x+5)(x-5)} \\
&= \frac{x}{x+5}
\end{aligned}$$

Oppgave 2

a)

$$\begin{aligned}
\lg(x \cdot y^2) - 2 \lg y + \lg\left(\frac{x}{y^2}\right) &= \lg x + 2 \lg y - 2 \lg y + \lg x - 2 \lg y \\
&= 2 \lg x - 2 \lg y \\
&= 2 \lg \frac{x}{y}
\end{aligned}$$

b)

$$\begin{aligned}
\lg \frac{1}{a^2} + 3 \lg a &= \lg 1 - 2 \lg a + 3 \lg a \\
&= \lg a
\end{aligned}$$

c)

$$\begin{aligned}
\lg(a^2b) - \lg\left(\frac{1}{ab}\right) &= \lg a^2 + \lg b - (\lg 1 - (\lg a + \lg b)) \\
&= 2 \lg a + \lg b + \lg a + \lg b \\
&= 3 \lg a + 2 \lg b
\end{aligned}$$

d)

$$\begin{aligned}
\lg(a^2b) + \lg(ab^2) + \lg\left(\frac{a}{b^3}\right) &= 2 \lg a + \lg b + \lg a + 2 \lg b + \lg a - 3 \lg b \\
&= 4 \lg a
\end{aligned}$$

e)

$$\begin{aligned}
\ln(a^2b) - 2 \ln a - \ln\left(\frac{1}{b}\right) &= 2 \ln a + \ln b - 2 \ln a - \ln 1 + \ln b \\
&= 2 \ln b
\end{aligned}$$

Oppgave 3

a)

$$\begin{aligned}
(x^2 \cdot e^{2x})' &= 2x \cdot e^{2x} + x^2 \cdot e^{2x} \cdot 2 \\
&= 2xe^{2x}(1+x)
\end{aligned}$$

b)

$$\begin{aligned}(x \cdot \ln x)' &= 1 \cdot \ln x + x \cdot \frac{1}{x} \\ &= \ln x + 1\end{aligned}$$

c)

$$\begin{aligned}(3e^{x^2+1})' &= 3e^{x^2+1} \cdot 2x \\ &= 6xe^{x^2+1}\end{aligned}$$

d)

$$\begin{aligned}(x^2 \cdot \ln x)' &= 2x \cdot \ln x + x^2 \cdot \frac{1}{x} \\ &= x(2 \ln x + 1)\end{aligned}$$

e)

$$\begin{aligned}\left(\frac{2x-1}{x+1}\right)' &= \frac{2(x+1) - (2x-1)}{(x+1)^2} \\ &= \frac{2x+2-2x+1}{(x+1)^2} \\ &= \frac{3}{(x+1)^2}\end{aligned}$$

Oppgave 4

a)

$$\begin{aligned}(xe^{-x})' &= 1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1) \\ &= x^{-x}(1-x)\end{aligned}$$

b)

$$\begin{aligned}(3e^{2x})' &= 3e^{2x} \cdot 2 \\ &= 6e^{2x}\end{aligned}$$

c)

$$\begin{aligned}((x^2+1)^4)' &= 4(x^2+1)^3 \cdot 2x \\ &= 8x(x^2+1)^3\end{aligned}$$

d)

$$\begin{aligned}(x \cdot e^{2x})' &= 1 \cdot e^{2x} + x \cdot e^{2x} \cdot 2 \\ &= e^{2x} + 2x \cdot e^{2x} \\ &= e^{2x}(1+2x)\end{aligned}$$

e)

$$(5e^{3x})' = 15e^{3x}$$

Oppgave 5

a)

$$\begin{aligned}
 (x^3 \cdot \ln(2x))' &= 3x^2 \cdot \ln(2x) + x^3 \cdot \frac{1}{2x} \cdot 2 \\
 &= 3x^2 \ln(2x) + x^2 \\
 &= x^2(3 \ln(2x) + 1)
 \end{aligned}$$

b)

$$(4e^{x^2-3x})' = 4e^{x^2-3x}(2x - 3)$$

c)

$$\begin{aligned}
 (2x \cdot e^x)' &= 2e^x + 2xe^x \\
 &= 2e^x(1 + x)
 \end{aligned}$$

d)

$$\begin{aligned}
 (3\sqrt{x^2-1})' &= (3(x^2-1)^{\frac{1}{2}})' \\
 &= 3(x^2-1)^{-\frac{1}{2}} \cdot 2x \\
 &= \frac{6x}{\sqrt{x^2-1}}
 \end{aligned}$$

e)

$$\begin{aligned}
 \left(\frac{500}{x} + 8x^2\right)' &= (500x^{-1} + 8x^2)' \\
 &= 500 \cdot (-1)x^{-2} + 16x \\
 &= -\frac{500}{x^2} + 16x
 \end{aligned}$$

Oppgave 6

a)

$$\begin{aligned}
 (3 \ln(2x))' &= 3 \cdot \frac{1}{2x} \cdot 2 \\
 &= \frac{3}{x}
 \end{aligned}$$

b)

$$(3x \cdot e^{x^2})' = 3 \cdot e^{x^2} + 3x \cdot e^{x^2} \cdot 2x$$

c)

$$(0,02x^3 + 0,6x^2 + 4,1)' = 0,06x^2 + 1,2x$$

d)

$$\begin{aligned}
 (\sqrt{x^2-1})' &= \frac{1}{2\sqrt{x^2-1}} \cdot 2x \\
 &= \frac{x}{\sqrt{x^2-1}}
 \end{aligned}$$

Oppgave 7

a)

$$\begin{aligned} \left(-\frac{1}{3}x^3 + 2x^2\right)' &= -x^2 + 4x \\ &= -x(x - 4) \end{aligned}$$

b)

$$\begin{aligned} ((x^2 + 3)^4)' &= 4(x^2 + 3) \cdot 2x \\ &= 8x(x^2 + 3) \end{aligned}$$

c)

$$\begin{aligned} (x^4 - 4x^3)' &= 4x^3 - 4 \cdot 3x^2 \\ &= 4x^2(x - 3) \end{aligned}$$

d)

$$\begin{aligned} (x^2 \cdot e^{2x})' &= 2x \cdot e^{2x} + x^2 \cdot e^{2x} \cdot 2 \\ &= 2xe^{2x}(1 + x) \end{aligned}$$

Oppgave 8

a)

$$(\pi r^2)' = 2\pi r$$

b)

$$\begin{aligned} \left(\frac{4}{3}\pi r^3\right)' &= \frac{4}{3} \cdot 3\pi r^2 \\ &= 4\pi r^2 \end{aligned}$$

c)

$$\begin{aligned} \left(\frac{2a^2}{e^a}\right)' &= \frac{4a \cdot e^a - 2a^2 \cdot e^a}{(e^a)^2} \\ &= \frac{e^a \cdot 2a(2 - a)}{(e^a)^2} \\ &= \frac{2a(2 - a)}{e^a} \end{aligned}$$

Oppgave 9