Eksamen R2
Vår 2020
Løsningsforslag

DEL 1

Oppgave 1 (3 poeng)

a) $f(x) = x \cdot \sin(x)$

$f'(x) = 1 \cdot \sin x + x \cdot \cos x = \underline{\underline{\sin x + x \cdot \cos x}}$

b) $g(x) = \frac{\cos(x^2)}{x}$

$g'(x) = \frac{-\sin(x^2) \cdot 2x \cdot x - \cos(x^2)}{x^2} = \underline{\underline{\frac{-2x^2 \sin(x^2) - \cos(x^2)}{x^2}}}$

Oppgave 2 (6 poeng)

a)

$$\int (x^2+3+e^{2x})\ dx = \underline{\underline{\frac{1}{3}x^3+3x+\frac{1}{2}e^{2x}+C}}$$

b) Substitusjon : $u = x^2$, $u' = 2x$, $dx = \frac{1}{2x}du$

$$\begin{aligned}\int (6x\cdot\sin(x^2))\ dx &= \int 6x\cdot\sin(u)\cdot\frac{1}{2x}\ du\\ &= 3\int \sin(u)\ du\\ &= -3\cos(u)+C\\ &= \underline{\underline{-3\cos(x^2)+C}}\end{aligned}$$

c) Delvis integral : $u = \ln x$, $v' = x$

$$\begin{aligned}\int x\cdot\ln\ dx &= \frac{1}{2}\cdot x^2\cdot\ln x - \int\frac{1}{2}x^2\cdot\frac{1}{x}\ dx\\ &= \frac{1}{2}\cdot x^2\cdot\ln x - \frac{1}{2}\int x\cdot\ dx\\ &= \frac{1}{2}\cdot x^2\ln x - \frac{1}{2}\cdot\frac{1}{2}\cdot x^2+C\\ &= \frac{1}{2}\cdot x^2(\ln x-\frac{1}{2})+C\end{aligned}$$

$$u = \ln x, u' = \frac{1}{x}$$

$$v = \frac{1}{2}x^2, v' = x$$

$$\begin{aligned}\int_1^e x\cdot\ln x\ dx &= \left[\frac{1}{2}\cdot x^2(\ln x-\frac{1}{2})\right]_1^e\\ &= \left(\frac{1}{2}\cdot e^2\left(\ln e-\frac{1}{2}\right)\right)-\left(\frac{1}{2}\cdot 1^2\left(\ln 1-\frac{1}{2}\right)\right)\\ &= \left(\frac{1}{2}\cdot e^2\left(\frac{1}{2}\right)\right)-\left(\frac{1}{2}\left(-\frac{1}{2}\right)\right)\\ &\underline{\underline{= \frac{1}{4}\left(e^2+1\right)}}\end{aligned}$$

Oppgave 3 (4 poeng)

a) Aritmetisk rekke

$a_1 = 3$

$a_n = a_1 + d(n-1) \rightarrow a_5 = 3 + d(5-1) = 3 + 4d$

$$\begin{aligned} s_n &= \frac{(a_1 + a_n)\cdot n}{2} \\ s_5 &= 55 \\ \frac{(3+3+4d)\cdot 5}{2} &= 55 \\ (3+2d)\cdot 5 &= 55 \\ 3+2d &= 11 \\ 2d &= 8 \\ d &= 4 \end{aligned}$$

$a_n = 3 + 4(n-1) = 3 + 4n - 4 = 4n - 1$

$s_{10} = \frac{(3+40-1)10}{2} = 42 \cdot 5 = \underline{\underline{210}}$

b)

$$7 + \frac{7}{2} + \frac{7}{4} + ...$$

$k = \frac{1}{2}$, rekka konvergerer fordi $|k| < 1$

$$S = \frac{7}{1-\frac{1}{2}} = 14$$

<u>Summen av rekka er 14.</u>

Oppgave 4 (5 poeng)

$$f(x) = 2 \cdot \sin(\pi x + \pi) - 1, x \in \big\langle -1, 3 \big\rangle$$

a) Toppunkter når $\sin(\pi(x+1)) = 1$:

$$\begin{aligned} \sin(\pi(x+1)) &= 1 \\ \pi(x+1) &= \frac{\pi}{2} + n \cdot 2\pi \\ x+1 &= \frac{1}{2} + 2n \\ x &= -\frac{1}{2} + 2n \\ x &= \left\{ -\frac{1}{2}, \frac{3}{2} \right\} \end{aligned}$$

Toppunkter :
$T_1 : (-\frac{1}{2}, f(-\frac{1}{2})) = \underline{\underline{(-\frac{1}{2}, 1)}}$

$T_2 = (\frac{3}{2}, f(\frac{3}{2})) = \underline{\underline{(\frac{3}{2}, 1)}}$

Bunnpunkter når $\sin(\pi(x+1)) = -1$:

$$\begin{aligned} \sin(\pi(x+1)) &= -1 \\ \pi(x+1) &= \frac{3\pi}{2} + n \cdot 2\pi \\ x+1 &= \frac{3}{2} + 2n \\ x &= \frac{1}{2} + 2n \\ x &= \left\{ \frac{1}{2}, \frac{5}{2} \right\} \end{aligned}$$

Bunnpunkter :
$B_1 : (\frac{1}{2}, f(\frac{1}{2})) = \underline{\underline{(\frac{1}{2}, -3)}}$

$B_2 = (\frac{5}{2}, f(\frac{5}{2})) = \underline{\underline{(\frac{5}{2}, -3)}}$

b) Skjæringspunkt mellom grafen og x-aksen : $y = 0$

$$\begin{aligned}
f(x) &= 0 \\
2 \cdot \sin(\pi x + \pi) - 1 &= 0 \\
2 \cdot \sin(\pi x + \pi) &= 1 \\
\sin(\pi x + \pi) &= \frac{1}{2} \\
\pi(x+1) = \frac{\pi}{6} + n \cdot 2\pi \vee \pi(x+1) &= \frac{5\pi}{6} + n \cdot 2\pi \\
x + 1 = \frac{1}{6} + 2n \vee x + 1 &= \frac{5}{6} + 2n \\
x = -\frac{5}{6} + 2n \vee x &= -\frac{1}{6} + 2n \\
x &= -\frac{5}{6}, -\frac{5}{6}, \frac{7}{6}, \frac{11}{6}
\end{aligned}$$

Skjæringspunkter med x-aksen : $\underline{\underline{(-\frac{5}{6}, 0), (-\frac{1}{6}, 0), (\frac{7}{6}, 0), (\frac{11}{6}, 0)}}$

Skjæringspunkt mellom grafen og y-aksen : $x = 0$

$$\begin{aligned}
f(0) &= 2\sin(\pi \cdot 0 + \pi) - 1 \\
&= -1
\end{aligned}$$

Skjæringspunkt med y-aksen : $\underline{\underline{(}}0, -1)$

Oppgave 5 (6 poeng)

a)

$$\begin{aligned}\overrightarrow{AB} &= [2-(-1), 2-3, 1-2] = [3,-1,-1]\\ \overrightarrow{AC} &= [0-(-1), 1-3, 0-2] = [1,-2,-2]\\ \overrightarrow{AC}\times\overrightarrow{AC} &= [-1(-2)-(-1)(-2), -(3(-2)-(-1)\cdot 1), 3(-2)-(-1)\cdot 1]\\ &= [2-2, -(-6+1), -6+1]\\ &= [0,5,-5]\end{aligned}$$

Som skulle vises.

b)

$$\overrightarrow{AT} = [5-(-1), 3-3, 8-2] = [6,0,6]$$

$$\begin{aligned}V &= \frac{1}{6}|(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AT}|\\ &= \frac{1}{6}|[0,5,-5]\cdot[6,0,6]|\\ &= \frac{1}{6}|0\cdot 6+5\cdot 0+(-5)\cdot 6|\\ &= \frac{1}{6}|-30|\\ &= 5\end{aligned}$$

<u>Volumet av pyramiden ABCT er 5.</u>

c) $\overrightarrow{AC}\times\overrightarrow{AC} = [0,5,-5] = 5[0,1,-1]$, da blir
$\vec{n} = [0,1,-1]$ er en normalvektor til planet som inneholder punktene A, B og C.

$$\begin{aligned}\alpha &: 0(x-0)+1(y-1)+(-1)(z-0) = 0\\ \alpha &: y-1-z = 0\\ \alpha &: y-z = 1\end{aligned}$$

Likningen for planet blir da : $\underline{\underline{\alpha : y-z=1}}$

Oppgave 6 (4 poeng)

a)

$$\begin{aligned}
-1 &< k < 1 \\
-1 &< \frac{\ln x}{2} < 1 \\
-2 &< \ln x < 2 \\
e^{-2} &< x < e^2
\end{aligned}$$

Rekka konvergerer når $\underline{\underline{x \in \langle e^{-2}, e^2 \rangle}}$

b)

$$\begin{aligned}
\frac{2}{1-\frac{\ln x}{2}} &= 4 \\
4(1-\frac{\ln x}{2}) &= 2 \\
4 - 2\ln x &= 2 \\
2\ln x &= 2 \\
\ln x &= 1 \\
x &= e
\end{aligned}$$

Summen av rekka blir 4 når $x = e$.

Oppgave 7 (2 poeng)

$$\begin{aligned}
2x \cdot y' - 3y &= 0 \\
y' &= \frac{3y}{2x}
\end{aligned}$$

Sjekker veksten (y') i alle 4 punktene :

$A(2,2) \quad : y' = \frac{6}{4} = \frac{3}{2}$
Passer ikke med tegningen der veksten er flat (y'=0)

$B(-2,2) \quad : y' = \frac{-6}{4} = -\frac{3}{2}$
Passer bra med tegningen.

$C(-2,-2)$
$: y' = \frac{-6}{-4} = \frac{3}{2}$
Passer bra med tegningen.

$D(2,-2) \quad : y' = \frac{6}{-4} = -\frac{3}{2}$
Passer ikke med tegningen, der veksten er ca. $-\frac{1}{3}$

Oppgave 8 (3 poeng)

$$\alpha : -2x + 2y - z = 21$$

$$\beta : -7x + 4y - 4z = 56$$

$P(-3, 7, -1)$, $Q(-4, 5, -2)$

$n_\alpha = [-2, 2, -1]$, $n_\beta = [-7, 4, -4]$

$$l_\alpha = \begin{cases} x = -3 - 2t \\ y = 7 + 2t \\ z = -1 - t \end{cases}$$

$$l_\beta = \begin{cases} x = -4 - 7s \\ y = 5 + 4s \\ z = -2 - 4s \end{cases}$$

Skjæringspunktet mellom linjene er senter i kula : Setter opp likningessettet :

$$\left\{ \begin{array}{l} -3 - 2t = -4 - 7s \Rightarrow 2t - 7s = 1 \\ 7 + 2t = 5 + 4s \Rightarrow 2t - 4s = -2 \\ -1 - t = -2 - 4s \Rightarrow t = 4s + 1 \end{array} \right\}$$

$2(4s + 1) - 7s = 1 \Rightarrow s = -1$

$t = 4 \cdot (-1) + 1 = -4 + 1 = -3$

Da blir senter i kulen : $S(-3 - 2(-3), 7 + 2(-3), -1 - (-3)) = (3, 1, 2)$

Radien til kula er :

$\overrightarrow{SP} = [-3 - 3, 7 - 1, -1 - 2] = [-6, 6, -3] = 3[-2, 2, -1]$

$r = 3 \cdot \sqrt{4 + 4 + 1} = 3 \cdot 3 = 9$

Likningen til kula blir da :$\underline{\underline{\text{K} : (x - 3)^2 + (y - 1)^2 + (z - 2)^2 = 9^2}}$

Oppgave 9 (3 poeng)

$a_1 = 2$

$a_n = a_{n-1} + n$

$a_n = \frac{n^2+n+2}{2}$

Bevis. Vi skal bevise at a_n er gyldig for alle $n \in \mathbb{N}$

P(1):$a_1 = \frac{1^2+1+2}{2} = \frac{4}{2} = 2$, altså sann for n=1.

P(n+1):

$$\begin{aligned} a_{n+1} &= a_n + (n+1) \\ &= \frac{n^2+n+2}{2} + (n+1) \\ &= \frac{n^2+n+2+2(n+1)}{2} \\ &= \frac{n^2+n+2+2n+2)}{2} \\ &= \frac{n^2+3n+4)}{2} \end{aligned}$$

$$\begin{aligned} a_{n+1} &= \frac{(n+1)^2+(n+1)+2}{2} \\ &= \frac{n^2+2n+1+n+3}{2} \\ &= \frac{n^2+3n+4}{2} \end{aligned}$$

Altså sann for (n+1), og dermed sann for alle $n \in \mathbb{N}$

■