Arbeidshefte

Faktorisering av uttrykk

Formler

Kvadratsetningene

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

Produkt / sum

$$(a+b)(a+c) = a^2 + a(b+c) + bc$$

$$(a+b)(a-c) = a^2 + a(b-c) + b \cdot (-c)$$

$$(a-b)(a-c) = a^2 + a(-b-c) + (-b)(-c)$$

Fullstendig kvadrat

$$\begin{aligned} x^2 + ax + b &= x^2 + ax + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b \\ &= (x + \left(\frac{a}{2}\right))^2 - \left(\frac{a}{2}\right)^2 + b \end{aligned}$$

Løse opp parenteser med 1.kvadratsetning

Formler

1.kvadratsetning

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\begin{aligned}(a+b)^2 &= (a+b)\cdot(a+b)\\ &= a\cdot a + a\cdot b + b\cdot a + a\cdot a\\ &= a^2 + ab + ba + a^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

Eksempler

$$\begin{aligned}(x+1)^2 &= (x+1)\cdot(x+1)\\ &= x\cdot x + x\cdot 1 + 1\cdot x + 1\cdot 1\\ &= x\cdot x + 2\cdot x\cdot 1 + 1^2\\ &= x^2 + 2x + 1\end{aligned}$$

$$\begin{aligned}(x+2)^2 &= (x+2)\cdot(x+2)\\ &= x\cdot x + x\cdot 2 + 2\cdot x + 2\cdot 2\\ &= x^2 + 2\cdot x\cdot 2 + 2^2\\ &= x^2 + 4x + 4\end{aligned}$$

$$\begin{aligned}(x+3)^2 &= (x+3)\cdot(x+3)\\ &= x\cdot x + x\cdot 3 + 3\cdot x + 3\cdot 3\\ &= x^2 + 2\cdot x\cdot 3 + 3^2\\ &= x^2 + 6x + 9\end{aligned}$$

Oppgave 1

Løs opp parentesene

1) $(x+6)^2 =$

2) $(x+7)^2 =$

3) $(x+8)^2 =$

4) $(x+9)^2 =$

5) $(x+10)^2 =$

6) $(x+11)^2 =$

7) $(x+12)^2 =$

8) $(x+13)^2 =$

Løse opp parenteser med 2.kvadratsetning

Formel

2.kvadratsetning

$$(a-b)^2 = a^2 - 2ab + b^2$$

Eksempler

$$\begin{aligned}(x-1)^2 &= (x-1)\cdot(x-1)\\ &= x\cdot x + x\cdot(-1) + (-1)\cdot x + (-1)\cdot(-1)\\ &= x^2 - x - x + (-1)^2\\ &= x^2 - 2\cdot x + (-1)^2\\ &= x^2 - 2x + 1\end{aligned}$$

$$\begin{aligned}(x-2)^2 &= (x-2)\cdot(x-2)\\ &= x\cdot x + x\cdot(-2) + (-2)\cdot x + (-2)\cdot(-2)\\ &= x^2 - 2x - 2x + (-2)^2\\ &= x^2 - 4\cdot x + (-2)^2\\ &= x^2 - 4x + 4\end{aligned}$$

$$\begin{aligned}(x-3)^2 &= (x-3)\cdot(x-3)\\ &= x\cdot x + x\cdot(-3) + (-3)\cdot x + (-3)\cdot(-3)\\ &= x^2 - 3x - 3x + (-3)^2\\ &= x^2 - 6\cdot x + (-3)^2\\ &= x^2 - 6x + 9\end{aligned}$$

Oppgave 2

Løs opp parentesene

1) $(x-1)^2 =$

2) $(x-2)^2 =$

3) $(x-3)^2 =$

4) $(x-4)^2 =$

5) $(x-5)^2 =$

6) $(x-6)^2 =$

7) $(x-7)^2 =$

8) $(x-8)^2 =$

Løse oppp parenteser med konjungatsetningen

Formel

Konjungatsetningen :

$$(a+b)(a-b) = a^2 - b^2$$

Eksempler

$$\begin{aligned}(x+1)(x-1) &= x \cdot x + x \cdot (-1) + 1 \cdot x + 1 \cdot (-1) \\ &= x^2 - x + x - 1 \\ &= x^2 - 1\end{aligned}$$

$$\begin{aligned}(x+2)(x-2) &= x \cdot x + x \cdot (-2) + 2 \cdot x + 2 \cdot (-2) \\ &= x^2 - 2x + 2x - 2^2 \\ &= x^2 - 4\end{aligned}$$

$$\begin{aligned}(x+3)(x-3) &= x \cdot x + x \cdot (-3) + 3 \cdot x + 3 \cdot (-3) \\ &= x^2 - 3x + 3x - 3^2 \\ &= x^2 - 9\end{aligned}$$

Oppgave 3

Løs opp parentesene

1) $(x+1)(x-1) =$

2) $(x+2)(x-2) =$

3) $(x+3)(x-3) =$

4) $(x+4)(x-4) =$

5) $(x+5)(x-5) =$

6) $(x+6)(x-6) =$

7) $(x+7)(x-7) =$

8) $(x+8)(x-8) =$

Løse opp parenteser

Formel

$$\begin{aligned}(a+b)(a+c) &= a\cdot a + a\cdot c + b\cdot a + b\cdot c \\ &= a^2 + ac + ab + bc \\ &= a^2 + a(b+c) + bc\end{aligned}$$

Eksempler

$$\begin{aligned}(x+1)(x+2) &= x^2 + (1+2)x + 1\cdot 2 \\ &= x^2 + 3x + 2\end{aligned}$$

$$\begin{aligned}(x+2)(x+5) &= x\cdot x + x\cdot 5 + 2\cdot x + 2\cdot 5 \\ &= x^2 + (2+5)x + 2\cdot 5 \\ &= x^2 + 7x + 10\end{aligned}$$

$$\begin{aligned}(x+1)(x-2) &= x^2 + (1-2)x + 1\cdot(-2) \\ &= x^2 - x - 2\end{aligned}$$

$$\begin{aligned}(x-2)(x-3) &= x^2 + (-2-3)x + (-2)\cdot(-3) \\ &= x^2 - 5x + 6\end{aligned}$$

Oppgave 4

Løs opp parentesene

1) $(x+1)(x+3) =$

2) $(x+2)(x+4) =$

3) $(x+3)(x+5) =$

4) $(x+4)(x+6) =$

5) $(x+5)(x+7) =$

6) $(x+6)(x+8) =$

7) $(x+7)(x+9) =$

8) $(x+8)(x+2) =$

9) $(x+9)(x+3) =$

10) $(x+10)(x+2) =$

Oppgave 5

Løs opp parentesene

1) $(x+1)(x-3) =$

2) $(x+2)(x-4) =$

3) $(x+3)(x-5) =$

4) $(x+4)(x-6) =$

5) $(x+5)(x-7) =$

6) $(x+6)(x-8) =$

7) $(x+7)(x-9) =$

8) $(x+8)(x-2) =$

9) $(x+9)(x-3) =$

10) $(x+10)(x-2) =$

Oppgave 6

Løs opp parentesene

1) $(x-1)(x-3) =$

2) $(x-2)(x-4) =$

3) $(x-3)(x-5) =$

4) $(x-4)(x-6) =$

5) $(x-5)(x-7) =$

6) $(x-6)(x-8) =$

7) $(x-7)(x-9) =$

8) $(x-8)(x-2) =$

9) $(x-9)(x-3) =$

10) $(x-10)(x-2) =$

Faktorisere med 1.kvadratsetning

Formel

1.kvadratsetning

$$\begin{aligned} a^2 + 2ab + b^2 &= a^2 + 2 \cdot a \cdot b + b^2 \\ &= (a+b)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 + 2x + 1 &= x^2 + 2 \cdot x \cdot 1 + 1^2 \\ &= (x+1)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 + 4x + 4 &= x^2 + 2 \cdot x \cdot 2 + 2^2 \\ &= (x+2)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 + 6x + 9 &= x^2 + 2 \cdot x \cdot 3 + 3^2 \\ &= (x+3)^2 \end{aligned}$$

Oppgave 7

Faktoriser uttrykket

1) $x^2 + 8x + 16 =$

2) $x^2 + 10x + 25 =$

3) $x^2 + 12x + 36 =$

4) $x^2 + 14x + 49 =$

5) $x^2 + 16x + 64 =$

6) $x^2 + 18x + 81 =$

7) $x^2 + 20x + 100 =$

8) $x^2 + 22x + 121 =$

9) $x^2 + 24x + 144 =$

10) $x^2 + 26x + 169 =$

Faktorisere med 2.kvadratsetning

Formel

2.kvadratsetning

$$\begin{aligned} a^2 - 2ab + b^2 &= a^2 - 2 \cdot a \cdot b + b^2 \\ &= (a-b)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 - 2x + 1 &= x^2 - 2 \cdot x \cdot 1 + 1^2 \\ &= (x-1)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 - 4x + 4 &= x^2 - 2 \cdot x \cdot 2 + 2^2 \\ &= (x-2)^2 \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 - 6x + 9 &= x^2 - 2 \cdot x \cdot 3 + 3^2 \\ &= (x-3)^2 \end{aligned}$$

Oppgave 8

Faktoriser uttrykket

1) $x^2 - 2x + 1 =$

2) $x^2 - 4x + 4 =$

3) $x^2 - 6x + 9 =$

4) $x^2 - 8x + 16 =$

5) $x^2 - 10x + 25 =$

6) $x^2 - 12x + 36 =$

7) $x^2 - 14x + 49 =$

8) $x^2 - 16x + 64 =$

9) $x^2 - 18x + 81 =$

10) $x^2 - 20x + 100 =$

Faktorisering med konjungatsetningen

Formel

Konjungatsetning

$$a^2 - b^2 = (a+b)(a-b)$$

Eksempler

$$\begin{aligned} x^2 - 1 &= x^2 - 1^2 \\ &= (x+1)(x-1) \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 - 4 &= x^2 - 2^2 \\ &= (x+2)(x-2) \end{aligned}$$

Oppgave 9

Faktoriser uttrykket

1) $x^2 - 1 =$

2) $x^2 - 4 =$

3) $x^2 - 9 =$

4) $x^2 - 16 =$

5) $x^2 - 25 =$

6) $x^2 - 36 =$

7) $x^2 - 49 =$

8) $x^2 - 64 =$

9) $x^2 - 81 =$

10) $x^2 - 100 =$

Faktorisere med produkt og sum

Formel

$$(a+b)(a+c) = a^2 + a(b+c) + bc$$

$$(a+b)(a-c) = a^2 + a(b-c) + b \cdot (-c)$$

$$(a-b)(a-c) = a^2 + a(-b-c) + (-b)(-c)$$

Eksempel

$$x^2 + 4x + 3 = x^2 + x \cdot (3+1) + (3 \cdot 1)$$

fordi $3 \cdot 1 = 3 \wedge 3 + 1 = 4$

$$x^2 + 4x + 3 = (x+1)(x+3)$$

Eksempler

$$\begin{aligned} x^2 - 3x - 4 &= x^2 + x \cdot (1-4) + (1 \cdot (-4)) \\ &= (x+1)(x-4) \end{aligned}$$

Eksempler

$$\begin{aligned} x^2 - 3x + 2 &= x^2 + x(-1-2) + ((-1) \cdot (-2)) \\ &= (x-1)(x-2) \end{aligned}$$

Oppgave 10

Faktoriser uttrykket

1) $x^2 + 4x + 3 =$

2) $x^2 + 6x + 8 =$

3) $x^2 + 8x + 15 =$

4) $x^2 + 10x + 24 =$

5) $x^2 + 12x + 35 =$

6) $x^2 + 14x + 48 =$

7) $x^2 + 16x + 63 =$

8) $x^2 + 10x + 16 =$

9) $x^2 + 12x + 27 =$

10) $x^2 + 12x + 20 =$

Oppgave 11

Faktoriser uttrykket

1) $x^2 - 2x - 3 =$

2) $x^2 - 2x - 8 =$

3) $x^2 - 2x - 15 =$

4) $x^2 - 2x - 24 =$

5) $x^2 - 2x - 35 =$

6) $x^2 - 2x - 48 =$

7) $x^2 - 2x - 63 =$

8) $x^2 + 6x - 16 =$

9) $x^2 + 6x - 27 =$

10) $x^2 + 8x - 20 =$

Oppgave 12

Faktoriser uttrykket

1) $x^2 - 4x + 3 =$

2) $x^2 - 6x + 8 =$

3) $x^2 - 8x + 15 =$

4) $x^2 - 10x + 24 =$

5) $x^2 - 12x + 35 =$

6) $x^1 - 14x + 48 =$

7) $x^2 - 16x + 63 =$

8) $x^2 - 10x + 16 =$

9) $x^2 - 12x + 27 =$

10) $x^2 - 12x + 20 =$

Faktorisere med fullstendig kvadrat

Eksempel

Fullstendig kvadrat

$$
\begin{aligned}
x^2 + 4x + 3 &= 0 \\
\text{Vi vet at} \quad (x+2)^2 &= x^2 + 4x + 4 \\
\text{altså et fullstendig kvadrat.} & \\
\text{setter da inn i likningen} \quad (+4-4) & \\
x^2 + 4x(+4-4) + 3 &= 0 \\
(x^2 + 4x + 4) + (-4+3) &= 0 \\
(x+2)^2 - 1 &= 0 \\
(x+2)^2 &= 1 \\
x + 2 &= \pm 1 \\
x = -3 \lor x &= -1
\end{aligned}
$$

Eksempler

$$
\begin{aligned}
x^2 + 4x - 5 &= x^2 + 4x(+4-4) - 5 \\
&= (x^2 + 4x + 4) + (-4-5) \\
&= (x+2)^2 - 3^2 \\
&= (x+2+3)(x+2-3) \\
&= (x+5)(x-1)
\end{aligned}
$$

Eksempler

$$
\begin{aligned}
x^2 + 2x - 3 &= x^2 + 2x + (1-1) - 3 \\
&= (x^2 + 2x + 1) + (-1-3) \\
&= (x+1)^2 - 2^2 \\
&= (x+1+2)(x+1-2) \\
&= (x+3)(x-1)
\end{aligned}
$$

Løs oppgave 10-12 på nytt, denne gangen med metoden fullstendige kvadrater.

Dette arbeidshefte :

Løsningsforslag :

14. januar 2024