

Arbeidshefte

Derivasjon R1

$$(a \cdot x^n)' = a \cdot n \cdot x^{n-1}$$

$$(a^x)' = a^x \cdot \ln a$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

Produktregelen

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

Kvotientregelen

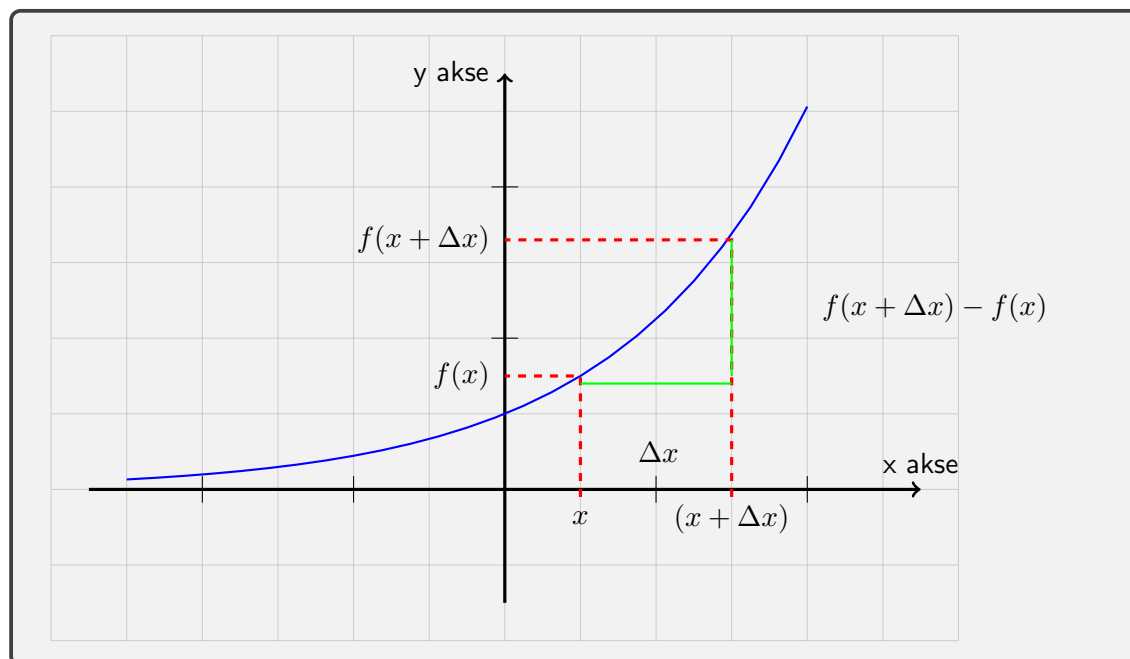
$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

Kjerneregelen

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Definisjon

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



Vi kan bruke definisjonen til å derivere :

$$\begin{aligned} f(x) &= x^2 \\ f(x + \Delta x) &= (x + \Delta x)^2 = x^2 + 2 \cdot x \cdot \Delta x + (\Delta x)^2 \\ f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x^2 + 2 \cdot x \cdot \Delta x + (\Delta x)^2) - (x^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\ &= 2x \end{aligned}$$

Heldigvis trenger vi ikke å bruke definisjonen, vi har regler.

Bruk definisjonen til å finne den deriverte :

Oppgave 1

$$f(x) = 2x$$

Oppgave 2

$$f(x) = x^2 + x$$

Derivasjonsregler

$$(a \cdot x^n)' = a \cdot n \cdot x^{n-1}$$

$$(\ln(x))' = \frac{1}{x}$$

$$(e^x)' = e^x$$

$$(a^x)' = \ln(a) \cdot a^x$$

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

Oppgave 3

1) $f(x) = 2x^2$

2) $f(x) = 2x$

3) $f(x) = 5$

Oppgave 4

1) $f(x) = e^x + \ln(x)$

2) $f(x) = 3 \cdot \ln(x)$

3) $f(x) = \frac{-4}{\sqrt{x}}$

4) $f(x) = \frac{1}{x^5}$

Produktregelen

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

Oppgave 5

1) $f(x) = x^2 \cdot e^x$

2) $f(x) = 2x^3 \cdot \ln(x)$

3) $f(x) = e^x \cdot (x^2 + 2)$

4) $f(x) = (x^2 + \pi)e^x$

5) $f(x) = x\sqrt{x}$

6) $f(x) = (x^2 + 3)x^3$

7) $f(x) = x \cdot (x^2 + 3)$

Kvotientregelen

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

Oppgave 6

1) $f(x) = \frac{x^2}{(x+2)}$

2) $f(x) = \frac{2x^3}{e^x}$

3) $f(x) = \frac{x+2}{x+1}$

4) $f(x) = \frac{2x}{\sqrt{x}}$

Oppgave 7

1) $f(x) = \frac{x^2+1}{x}$

2) $f(x) = \frac{x^2-x-6}{x+2}$

3) $f(x) = \frac{e^x}{\ln(x)}$

4) $f(x) = \frac{e^x}{x^2}$

Kjernerregelen

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Oppgave 8

1) $f(x) = \ln(x^2)$

2) $f(x) = (2x)^3$

3) $f(x) = (x + 2)^4$

4) $f(x) = e^{x^2}$

5) $f(x) = \sqrt{2x}$

Oppgave 9

1) $f(x) = 2\sqrt{x^3}$

2) $f(x) = (2x^2 - 4)^3$

3) $f(x) = (x^2 - 4x + 3)^2$

4) $f(x) = \sqrt{x^2 - 4}$

5) $f(x) = \frac{2x^2}{(3x-2)^2}$

6) $f(x) = x \cdot \ln(x^2 + 3)$

Oppgave 10

1) $f(x) = x^3 - 2x + \ln(2x)$

2) $f(x) = x^4 - 4x + 17$

3) $f(x) = (1 - x)^3 - \ln(1 - x)$

4) $f(x) = -\ln(1 - x)^2$

5) $f(x) = (3x^2 + 1)^5$

6) $f(x) = (x - 1)^3$

Oppgave 11

1) $f(x) = (1 - \sqrt{x})^3$

2) $f(x) = (1 + x + x^2)^6$

3) $f(x) = \sqrt{x^2 - 4}$

4) $f(x) = \sqrt{2x - x^4}$

5) $f(x) = \ln(\sqrt{1 - x})$

6) $f(x) = e^{3x+1}$

Oppgave 12

1) $f(x) = e^{1-x}$

2) $f(x) = \frac{x+1}{(x-2)^4}$

3) $f(x) = \sqrt{x} - x - x^2$

4) $f(x) = x\sqrt{x} + x$

5) $f(x) = x(x+1)^2$

6) $f(x) = \frac{x^2-1}{x^3}$

Oppgave 13

1) $f(x) = \frac{x^2}{x-4}$

2) $f(x) = \frac{x+1}{x-2}$

3) $f(x) = \sqrt{x^2 - 9}$

4) $f(x) = (2x - 1)^5$

5) $f(x) = e^{x+3}$

6) $f(x) = \ln x^4$

Oppgave 14

1) $f(x) = 5e^{3-8x}$

2) $f(x) = \ln x^3 - 1$

3) $f(x) = x^2 \cdot \ln x$

FASIT

Oppgave 1

1) 2

Oppgave 2

1) $2x + 1$

Oppgave 3

1) $4x$

3) 0

2) 2

Oppgave 4

1) $e^x + \frac{1}{x}$

3) $\frac{2}{x\sqrt{x}}$

2) $\frac{3}{x}$

4) $-\frac{5}{x^6}$

Oppgave 5

1) $xe^x(x + 2)$

5) $\frac{3\sqrt{x}}{2}$

2) $2x^2(3\ln(x) + 1)$

6) $x^2(5x^2 + 9)$

3) $e^x(x^2 + 2x + 2)$

7) $3(x^2 + 1)$

4) $e^x(x^2 + 2x + \pi)$

Oppgave 6

1) $\frac{x(x+4)}{(x+2)^2}$

3) $\frac{-1}{(x+1)^2}$

2) $\frac{2x^2(3-x)}{e^x}$

4) $\frac{1}{\sqrt{x}}$

Oppgave 7

1) $\frac{(x+1)(x-1)}{x^2}$

3) $\frac{e^x(\ln(x) - \frac{1}{x})}{(\ln(x))^2}$

2) 1

4) $\frac{e^x(x-2)}{x^3}$

Oppgave 8

1) $\frac{2}{x}$

2) $24x^2$

3) $4(x+2)^3$

4) $2x \cdot e^{x^2}$

5) $\frac{1}{\sqrt{2x}}$

Oppgave 9

1) $3\sqrt{x}$

2) $12x(2x^2 - 4)^2$

3) $4(x-3)(x-2)(x-1)$

4) $\frac{x}{\sqrt{x^2-4}}$

5) $\frac{-8x}{(3x-2)^3}$

6) $\ln(x^2 + 3) + \frac{2x^2}{x^2+3}$

Oppgave 10

1) $3x^2 - 2 + \frac{1}{x}$

2) $4(x^3 - 1)$

3) $-3(1-x)^2 + \frac{1}{x-1}$

4) $\frac{2}{x-1}$

5) $30x(3x^2 + 1)^4$

6) $3(x-1)^2$

Oppgave 11

1) $-\frac{3(1-\sqrt{x})^2}{2\sqrt{x}}$

2) $6(x^2 + x + 1)^5(2x + 1)$

3) $\frac{x}{\sqrt{x^2-4}}$

4) $-\frac{2x^3-1}{\sqrt{2x-x^4}}$

5) $\frac{-1}{\sqrt{1-x}}$

6) $3e^{3x+1}$

Oppgave 12

1) $-e^{1-x}$

2) $-\frac{3(x+2)}{(x-2)^5}$

3) $\frac{1}{2\sqrt{x}} - 1 - 2x$

4) $\frac{3}{2}\sqrt{x} + 1$

5) $(x+1)(3x+1)$

6) $\frac{-x^2+3}{x^4}$

Oppgave 13

1) $\frac{x(x-8)}{(x-4)^2}$

2) $-\frac{3}{(x-2)^2}$

3) $\frac{x}{\sqrt{x^2-9}}$

4) $10(2x-1)^4$

5) e^{x+3}

6) $\frac{4}{x}$

Oppgave 14

1) $-40e^{3-8x}$

2) $\frac{3}{x}$

3) $x(2\ln(x) + 1)$