

Arbeidshefte Integralregning Metoder

$$\int f(g(x)) dx = \int f(u) \cdot \frac{1}{u'} du, \quad u = g(x)$$

$$\int u' \cdot v dx = u \cdot v - \int u \cdot v' dx$$

$$\int \frac{1}{a \cdot b} dx = \int \frac{A}{a} + \frac{B}{b} dx$$

Integrasjon ved substitusjon

$$\int f(g(x)) dx = \int f(u) \cdot \frac{1}{u'} du, \quad u = g(x)$$

$$\int e^{2x} dx = \int e^u \frac{1}{2} du = \frac{1}{2} \int e^u du = \frac{1}{2} e^{2x} + C$$
$$u = 2x, \quad u' = 2, \quad dx = \frac{1}{u'} du = \frac{1}{2} du$$

Oppgave 1

1) $\int e^{5x} dx$

2) $\int \frac{2 \ln x}{x} dx =$

3) $\int (2x + 1)e^{x^2+x} dx =$

4) $\int \frac{x}{x^2+1} dx =$

5) $\int \frac{e^x}{1+e^x} dx =$

Oppgave 2

1) $\int \frac{(\ln x)^2}{x} dx =$

2) $\int x \cdot \cos(x^2 + 1) dx =$

3) $\int x \cdot (x^2 + 2)^6 dx =$

4) $\int 2x e^{x^2+1} dx =$

5) $\int \frac{\cos x}{\sin x+1} dx =$

6) $\int 4e^{2x+1} dx =$

7) $\int 6\pi \sin(2\pi x) dx =$

8) $\int \frac{1}{3x+1} dx =$

Oppgave 3

1) $\int \frac{8x}{2x^2+5} dx =$

2) $\int \frac{6x^2}{x^3+1} dx =$

3) $\int \frac{2x}{(x^2+3)^3} dx =$

4) $\int x \cdot (x^2 + 1)^2 dx =$

5) $\int \cos(x) \cdot e^{\sin x} dx =$

6) $\int dx =$

Integrasjon ved delvis integral

$$\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx$$

$$\begin{aligned}\int 4x \cdot e^x \, dx &= 4x \cdot e^x - \int 4 \cdot e^x \, dx \\ &= 4xe^x - 4e^x + C \\ &= 4e^x(x - 1) + C\end{aligned}$$

$$\begin{aligned}u &= 4x, \quad u' = 4 \\ v &= e^x, \quad v' = e^x\end{aligned}$$

Oppgave 4

1) $\int 2x \cdot \ln(x) \, dx =$

2) $\int x^4 \cdot \ln(x) \, dx =$

3) $\int \cos x \cdot x \, dx =$

4) $\int \ln(x) \, dx =$

5) $\int (2x + 1) \cdot \sin x \, dx =$

Oppgave 5

1) $\int 2x \cdot e^x dx =$

2) $\int x \cdot \cos x dx =$

3) $\int e^x(x^2 - 1) dx =$

4) $\int (x^2 + 2x + 1)e^x dx =$

5) $\int 8x^2 \cdot e^{2x} dx =$

6) $\int \cos x \cdot e^x dx =$

7) $\int e^x \cdot x^2 dx =$

8) $\int \sin x \cdot e^x dx =$

Integrasjon ved delbrøkkopp spalting

$$\int \frac{1}{a \cdot b} dx = \int \frac{A}{a} + \frac{B}{b} dx$$

Eksempel

$$\begin{aligned} \int \frac{2}{x^2 - 1} dx &= \int \frac{2}{(x+1)(x-1)} dx \\ &= \int \frac{A}{x+1} + \frac{B}{x-1} dx \\ (\text{forts. etter mellomregn.}) &= \int \frac{1}{x+1} - \frac{1}{x-1} dx \\ &= \ln|x-1| - \ln|x+1| + C \\ &= \ln \left| \frac{x-1}{x+1} \right| + C \end{aligned}$$

Mellomregning

$$\begin{aligned} \frac{2}{(x+1)(x-1)} &= \frac{A}{x+1} + \frac{B}{x-1} \quad | \cdot \text{FN} \\ 2 &= A(x-1) + B(x+1) \\ x = -1 : 2 &= -2A \Rightarrow A = -1 \\ x = 1 : 2 &= 2B \Rightarrow B = 1 \end{aligned}$$

Oppgave 6

1) $\int \frac{1}{x^2-1} dx =$

2) $\int \frac{4}{x^2-x} dx =$

Oppgave 7

1) $\int \frac{1}{x^2-4} dx =$

2) $\int \frac{1}{2x^2-18} dx =$

3) $\int \frac{1}{x+1} + \frac{1}{x+2} dx =$

4) $\int \frac{2}{x^2-9} dx =$

5) $\int \frac{1}{x^2-16} dx =$

6) $\int \frac{5x+1}{x^2+x-2} dx =$

Oppgave 8

1) $\int \frac{3x+1}{x^2-x-6} dx =$

2) $\int \frac{x^2+x+13}{x^3-2x^2-5x+6} dx =$

3) $\int \frac{8}{x^3-4x} dx =$

4) $\int \frac{x+4}{x^2+2x} dx =$

5) $\int \frac{3x}{x^2-x-2} dx =$

6) $\int \frac{2x+4}{x^2+4x+3} dx =$

Oppgave 9

1) $\int \frac{3x-5}{x^2-x-12} dx =$

2) $\int \frac{x^2+x-1}{x^2-x} dx =$

3) $\int \frac{2x^2+5x+1}{x^2+x} dx =$

4) $\int \frac{2x^3+x^2-2x-3}{x^2-1} dx =$

Sinus og cosinus

Oppgave 10

1) $\int 2 \cdot \cos x \, dx =$

2) $\int -\sin(3x - \pi) \, dx =$

3) $\int 2 \cdot \cos(2x - 24) + 12 \, dx =$

4) $\int 4 \cos(2x) \, dx =$

5) $\int \frac{1}{2} \sin(\pi \cdot x) \, dx =$

6) $\int \sin x \cdot \cos x \, dx =$

7) $\int \sin^2 x \, dx =$

Oppgave 11

1) $\int (3x^2 + 3x + \pi) dx =$

2) $\int \frac{1}{2}\sqrt{x} + \frac{2}{x} dx =$

3) $\int e^{x^2+x}(2x + 1) dx =$

4) $\int e^{2x} \cdot 2x dx =$

5) $\int e^{x^2} \cdot 2x dx =$

6) $\int \frac{2x}{x^2-4} dx =$

7) $\int \frac{2x+4}{x^2-x} dx =$

8) $\int 10x(x^2 + 1)^4 dx =$

Oppgave 12

1) $\int (4x + 4)e^{x^2+2x+1} dx =$

2) $\int \frac{e^x}{e^x+1} dx =$

3) $\int \tan x dx =$

4) $\int (2x - \cos x) dx =$

5) $\int e^x + 3x^2 dx =$

6) $\int \frac{1}{1+\sqrt{x}} dx =$

7) $\int x \ln(x) dx =$

8) $\int 2x \cdot \sin^2 x dx =$

FASIT

Oppgave 1

1) $\frac{1}{5}e^{5x} + C$

4) $\frac{1}{2} \ln(x^2 + 1) + C$

2) $(\ln x)^2 + C$

5) $\ln(1 + e^x) + C$

3) $e^{x^2+x} + C$

Oppgave 2

1) $\frac{1}{3}(\ln x)^3 + C$

5) $\ln |\sin x + 1| + C$

2) $\frac{1}{2} \sin(x^2 + 1) + C$

6) $2e^{2x+1} + C$

3) $\frac{1}{14}(x^2 + 2)^7 + C$

7) $-3 \cos(2\pi x) + C$

4) $e^{x^2+1} + C$

8) $\frac{1}{3} \ln |3x + 1| + C$

Oppgave 3

1) $2 \ln |2x^2 + 5| + C$

4) $\frac{1}{6}(x^2 + 1)^3 + C$

2) $2 \ln |x^3 + 1| + C$

5) $e^{\sin x} + C$

3) $-\frac{1}{2(x^2+3)^2} + C$

6) C

Oppgave 4

1) $x^2(\ln x - \frac{1}{2}) + C$

4) $x \cdot \ln x - x + C$

2) $\frac{1}{5}x^5(\ln x - \frac{1}{5}) + C$

5) $-\cos x(2x + 1) + 2 \sin x + C$

3) $x \cdot \sin x + \cos x + C$

Oppgave 5

1) $2e^x(x - 1) + C$

5) $2e^{2x}(2x^2 - 2x + 1) + C$

2) $x \cdot \sin x + \cos x + C$

6) $\frac{1}{2}e^x(\cos x + \sin x) + C$

3) $e^x(x - 1)^2 + C$

7) $e^x(x^2 - 2x + 2) + C$

4) $e^x(x^2 + 1) + C$

8) $\frac{1}{2}e^x(\sin x - \cos x) + C$

Oppgave 6

1) $\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C$

2) $4 \ln \left| \frac{x-1}{x} \right| + C$

Oppgave 7

1) $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$

4) $\frac{1}{3} \ln \left| \frac{x-3}{x+3} \right| + C$

2) $\frac{1}{12} \ln \left| \frac{x-3}{x+3} \right| + C$

5) $\frac{1}{8} \ln \left| \frac{x-4}{x+4} \right| + C$

3) $\ln |(x+1)(x+2)| + C$

6) $3 \ln |x+2| + 2 \ln |x-1| + C$

Oppgave 8

1) $\ln |x+2| + 2 \ln |x-3| + C$

4) $2 \ln |x| - \ln |x+2| + C$

2) $\ln |x+2| + \frac{5}{2} \ln \left| \frac{x-3}{x-1} \right| + C$

5) $2 \ln |x-2| + \ln |x+1| + C$

3) $\ln |x^2 - 4| - \ln |x^2| + C$

6) $\ln |(x+1)(x+3)| + C$

Oppgave 9

1) $2 \ln |x+3| + \ln |x-4| + C$

3) $\ln |x| + 2 \ln |x+1| + 2x + C$

2) $\ln |x| + \ln |x-1| + x + C$

4) $x^2 + x + \ln |x+1| - \ln |x-1| + C$

Oppgave 10

1) $2 \sin x + C$

5) $-\frac{1}{2\pi} \cos(\pi x) + C$

2) $\frac{1}{3} \cos(3x - \pi) + C$

6) $\frac{1}{2} \sin^2 x + C$

3) $\sin(2x - 24) + 12x + C$

4) $2 \sin(2x) + C$

7) $-\frac{1}{4} \sin(2x) + \frac{1}{2}x + C$

Oppgave 11

1) $x^3 + \frac{3}{2}x^2 + \pi x + C$

5) $e^{x^2} + C$

2) $\frac{1}{3}x\sqrt{x} + 2 \ln |x| + C$

6) $\ln |x^2 - 4| + C$

3) $e^{x^2+x} + C$

7) $6 \ln |x-1| - 4 \ln |x| + C$

4) $\frac{1}{2}(2x-1)e^{2x} + C$

8) $(x^2+1)^5 + C$

Oppgave 12

1) $2e^{(x+1)^2} + C$

5) $x^3 + e^x + C$

2) $\ln|e^x + 1| + C$

6) $2\sqrt{x} - 2\ln|\sqrt{x} + 1| + C$

3) $-\ln|\cos x| + C$

4) $x^2 - \sin x + C$

7) $\frac{1}{2}x^2(\ln|x| - \frac{1}{2}) + C$