Arbeidshefte
Vektorer - R1
Løsningsforslag

Oppgave 2

1) $[2,3] = 2\vec{e_x} + 3\vec{e_y}$
2) $[2,-2] = 2\vec{e_x} - 2\vec{e_y}$
3) $[-1,3] = -\vec{e_x} + 3\vec{e_y}$
4) $[1,-2] = \vec{e_x} - 2\vec{e_y}$
5) $[4,-5] = 4\vec{e_x} - 5\vec{e_y}$

Oppgave 3

1) $2\vec{e_x} - \vec{e_y} = [2,-1]$
2) $3\vec{e_x} + 2\vec{e_y} = [3,2]$
3) $\vec{e_x} - 2\vec{e_y} = [1,-2]$
4) $-\vec{e_x} - \vec{e_y} = [-1,-1]$
5) $5\vec{e_x} + 7\vec{e_y} = [5,7]$

Oppgave 4

1) $[4,2] = k \cdot [2,1] \Rightarrow 4 = 2k \wedge 2 = k \Rightarrow k = 2$
2) $[6,8] = k \cdot [3,4] \Rightarrow 6 = 3k \wedge 8 = 4k \Rightarrow k = 2$
3) $k \cdot [1,3] = [3,9] \Rightarrow k = 3 \wedge 3k = 9 \Rightarrow k = 3$
4) $[3,1] = k \cdot [6,2] \Rightarrow 3 = 6k \wedge 1 = 2k \Rightarrow k = \frac{1}{2}$

Oppgave 5

1) $x = \frac{1}{2}, y = -\frac{2}{3}$
2) $(x+1) = 5 \Rightarrow x = 4$
   $y - 1 = -3 \Rightarrow y = -2$
3) $\frac{x}{2} = \frac{3}{2} \Rightarrow x = 3$
   $-y = 2 \Rightarrow y = -2$
4) $x^2 = 2 \Rightarrow x = \pm 2$
   $\frac{y}{3} = -9 \Rightarrow y = -27$

Oppgave 6

1) $\vec{a} = [1, 1]$

2) $\vec{b} = [-2, 3]$

3) $\vec{c} = [-1, 4]$

4) $\vec{d} = [-3, -1]$

5) $-\vec{d} = -[-3, -1] = [3, 1]$

Oppgave 7

1) $(0, 0), (3, 2) \Rightarrow [3 - 0, 2 - 0] = [3, 2]$

2) $(1, 2), (5, -2) \Rightarrow [5 - 1, -2 - 2] = [4, -4]$

3) $(1, 3), (-2, 5) \Rightarrow [-2 - 1, 5 - 3] = [-3, 2]$

4) $(-4, -3), (-2, 3) \Rightarrow [-2 + 4, 3 + 3] = [2, 6]$

5) $(2, -2), (2, 2) \Rightarrow [2 - 2, 2 + 2] = [0, 4]$

6) $(5, 0), (1, 5) \Rightarrow [1 - 5, 5 - 0] = [-4, 5]$

7) $(-1, -3), (-2, 1) \Rightarrow [-2 - (-1), 1 - (-2)] = [-1, 3]$

8) $(-5, 3), (-4, 0) \Rightarrow [-4 - (-5), 0 - 3] = [1, -3]$

Oppgave 8

1) $[1, 2] + [4, 2] = [1 + 4, 2 + 2] = [5, 4]$

2) $[4, -1] + [2, 6] = [4 + 2, -1 + 6] = [6, 5]$

3) $[-5, -2] + [5, -1] = [-5 + 5, -2 - 1] = [0, -3]$

4) $[0, 2] + [0, 3] = [0 + 0, 2 + 3] = [0, 5]$

5) $[-4, 0] - [4, 5] = [-4 - 4, 0 - 5] = [-8, -5]$

6) $[1, 1] - [5, -1] = [1 - 5, 1 + 1] = [-4, 2]$

7) $[2, -4] - [-4, 2] = [2 + 4, -4 - 2] = [6, -6]$

8) $[4, 5] - [1, 1] = [4 - 1, 5 - 1] = [3, 4]$

Oppgave 9

1) $|[2,3]| = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$
2) $|[1,3]| = \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}$
3) $|[-2,3]| = \sqrt{(-2)^2+3^2} = \sqrt{4+9} = \sqrt{13}$
4) $|[1,-4]| = \sqrt{1^2+(-4)^2} = \sqrt{1+16} = \sqrt{17}$
5) $|[2,-1]| = \sqrt{2^2+(-1)^2} = \sqrt{4+1} = \sqrt{5}$
6) $|[-3,-2]| = \sqrt{(-3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$
7) $|[3,4]| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$
8) $|[4,5]| = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
9) $|[1,1]| = \sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$
10) $|[2,1]| = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

Oppgave 10

1) $[1,2] \cdot [4,2] = 1 \cdot 4 + 2 \cdot 2 = 4 + 4 = 8$
2) $[4,-1] \cdot [2,6] = 4 \cdot 2 + (-1) \cdot 6 = 8 - 6 = 2$
3) $[-5,-2] \cdot [5,-1] = -5 \cdot 5 + (-2) \cdot (-1) = -25 + 2 = -23$
4) $[0,2] \cdot [0,3] = 0 \cdot 0 + 2 \cdot 3 = 6$
5) $[-4,0] \cdot [4,5] = -4 \cdot 4 + 0 \cdot 5 = -16$
6) $[1,1] \cdot [5,-1] = 1 \cdot 5 + 1 \cdot (-1) = 5 - 1 = 4$
7) $[2,-4] \cdot [-4,2] = 2 \cdot (-4) + (-4) \cdot 2 = -8 - 8 = -16$
8) $[4,5] \cdot [1,1] = 4 \cdot 1 + 5 \cdot 1 = 4 + 5 = 9$

Oppgave 11

1) $\cos\alpha = \frac{30}{12 \cdot 5} = \frac{30}{60} = \frac{1}{2}$, $\alpha = 60^o$
2) $|\vec{a}| = \sqrt{13}$
   $|\vec{b}| = \sqrt{9+4} = \sqrt{13}$
   $\vec{a} \cdot \vec{b} = -6 + 6 = 0$
   $\cos\alpha = 0$
   $\alpha = 90^o$
3) $|\vec{a}| = 3$
   $|\vec{b}| = 3$
   $\vec{a} \cdot \vec{b}0$
   $\cos\alpha = 0$
   $\alpha = 90^o$

4) $|\vec{a}| = \sqrt{2}$
$|\vec{b}| = \sqrt{2}$
$\vec{a} \cdot \vec{b}0$
$\cos\alpha = 0$
$\alpha = 90^o$

Oppgave 12

1) $[-5,-2] \cdot [5,-2] = -25 + 4 \neq 0$, ikke orthogonale

2) $[0,2] \cdot [3,0] = 0 + 0 = 0$, orthogonale

3) $[-4,0] \cdot [4,5] = -16 + 0 \neq 0$, ikke orthogonale

4) $[1,5] \cdot [5,-1] = 5 - 5 = 0$, orthogonale

5) $[2,-4] \cdot [4,2] = 8 - 8 = 0$, orthogonale

6) $[4,5] \cdot [1,1] = 4 + 5 \neq 0$, ikke orthogonale

Oppgave 13

1) $k\vec{a} = \vec{b} \Rightarrow k[2,1] = [4,2] \Rightarrow 2k = 4, k = 2$, parallelle

2) $[3,2] = k[1,-2] \Rightarrow k = 3$ og $-2k = 2$, ikke parallelle

3) $k[1,-4] = [-2,8] \Rightarrow k = -2$ og $-4k = 8 \Rightarrow k = -2$, parallelle

4) $k[3,-4] = [-3,2] \Rightarrow 3k = -3$ og $-4k = 2$, ikke parallelle

Oppgave 14

1) $k[1,x] = [2,-1] \Rightarrow k = 2 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$

2) $[4x,4] = [16,x] \Rightarrow x = 4$

3) $k[-2,-3] = [4,x] \Rightarrow -2k = 4 \Rightarrow k = -2 \Rightarrow x = 6$

4) $[x,4] = k[3,2] \Rightarrow k = 2 \Rightarrow x = 6$

5) $k[x,1] = [4,2] \Rightarrow k = 2 \Rightarrow x = 2$

Oppgave 15

1) $\overrightarrow{AB} = [0-3, 2-1] = [-3,1]$
$\overrightarrow{AC} = [2-3, 5-1] = [-1,4]$

2) $\overrightarrow{AB} = \cdot\overrightarrow{AC} = [-3,1] \cdot [-1,4] = 3 + 4 = 7$

3) $|\overrightarrow{AB}| = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$
$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$

Oppgave 16

1) C ligger på y-aksen, da må x-koordinatet null, $C(0, y)$
$\overrightarrow{AB} = [5-2, -2-3] = [3, -5]$
$\overrightarrow{AC} = [0-2, y-3] = [-2, (y-3)]$

Dersom $\overrightarrow{AB} \perp \overrightarrow{AC}$ så er $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$\overrightarrow{AB} = [3, -5], C = (0, y), \overrightarrow{AC} = [-3, y+5]$

$$\begin{aligned}
\overrightarrow{AB} &\perp \overrightarrow{AC} \\
\overrightarrow{AB} \cdot \overrightarrow{AC} &= 0 \\
[3, -5] \cdot [-3, y+5] &= 0 \\
-9 - 5(y+5) &= 0 \\
y &= \frac{9}{5} \\
x &= \left(0, \frac{9}{5}\right)
\end{aligned}$$

2) D ligger på x-aksen, altså $D = (x, 0)$

$$\begin{aligned}
\overrightarrow{AC} \cdot \overrightarrow{CD} &= 0 \\
[-2, \frac{9}{5} - 3] \cdot [x, 0 - \frac{9}{5}] &= 0 \\
[-2, \frac{9-15}{5}] \cdot [x, -\frac{9}{5}] &= 0 \\
-2x + \frac{-6}{5} \cdot \frac{-9}{5} &= 0 \\
2x &= \frac{54}{25} \\
x &= \frac{27}{25}
\end{aligned}$$

Altså er $D = (\frac{27}{25}, 0)$

Oppgave 17 - ExR1V13 del1

a)

$$\begin{aligned}
\vec{a} &\perp \vec{b} \\
\vec{a} \cdot \vec{b} &= 0 \\
[2, 3] \cdot [-6, 4] &= -12 + 12 \\
&= 0
\end{aligned}$$

Altså er de orthogonale.

b)

$$\begin{aligned}
\vec{c} &= k \cdot \vec{a} + t \cdot \vec{b} \\
[3, 11] &= k \cdot [2, 3] + t[-6, 4] \\
3 = 2k - 6t \wedge 11 &= 3k + 4t \\
9 = 6k - 18t \wedge 22 &= 6k + 8t \\
13 &= 26t \\
t &= \frac{13}{26} = \frac{1}{2} \\
2k &= 3 + 6t = 3 + 3 = 6 \\
k &= 3
\end{aligned}$$

Oppgave 18 - ExR1H13 del1

Vektorene $\vec{a} = [1, 3]$, $\vec{b} = [3, 2]$ og $\vec{c} = [-1, 2]$ er gitt.

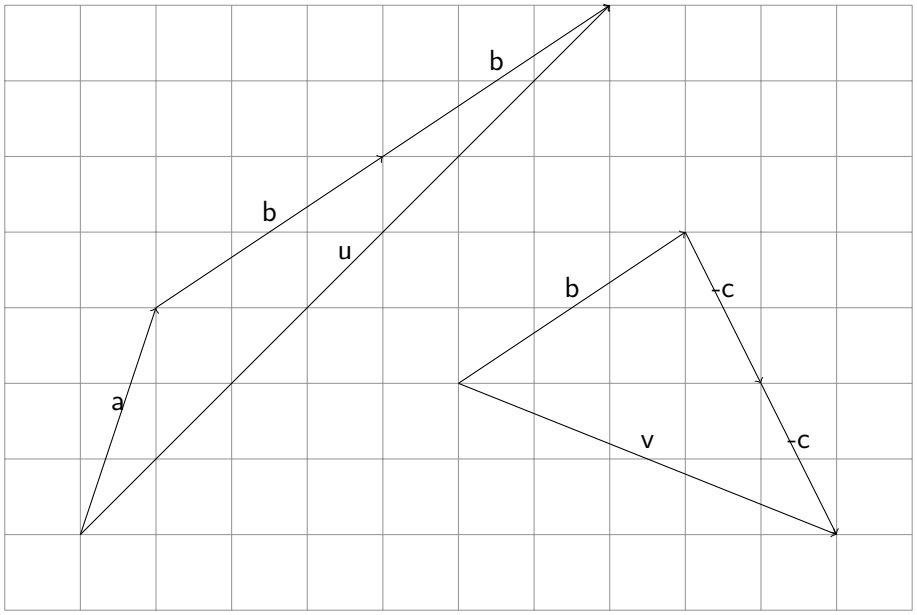
a) $\vec{u} = \vec{a} + 2\vec{b} = [1, 3] + 2[3, 2] = [1 + 2 \cdot 3, 3 + 2 \cdot 2] = [7, 7]$

$\vec{v} = \vec{b} - 2\vec{c} = [3, 2] - 2[-1, 2] = [3 - 2 \cdot (-1), 2 - 2 \cdot 2] = [5, -2]$

b)

$$\begin{aligned}
\vec{u} &\perp \vec{v} \\
\vec{u} \cdot \vec{v} &= 0 \\
[7, 7] \cdot [5, -2] &\neq 0
\end{aligned}$$

altså ikke orthogonale.



Oppgave 19 - (ExR1V14 del1, oppg.3, 4 poeng)

Vektorene $\vec{a} = [-2, 1]$, $\vec{b} = [3, 6]$ og $\vec{c} = [k - 1, 4]$ er gitt.

a) $-2\vec{a} + \vec{b} = -2[-2, 1] + [3, 6] = [4 + 3, -2 + 6] = [7, 4]$
$\vec{a} \cdot \vec{b} = [-2, 1] \cdot [3, 6] = -6 + 6 = 0$

b) $\vec{a} || \vec{c} \Rightarrow \vec{a} = k \cdot \vec{c} \Rightarrow t[-2, 1] = [k - 1, 4]$
$t = 4 \Rightarrow -2t = k - 1 \Rightarrow -2 \cdot 4 = k - 1 \Rightarrow k = -7$

c) $|\vec{c}| = |2\vec{a}|$
$|\vec{c}| = \sqrt{(k-1)^2 + 4^2} = \sqrt{k^2 - 2k + 17}$
$|2\vec{a}| = 2|\vec{a}| = 2 \cdot \sqrt{(-2)^2 + 1^2} = 2\sqrt{4+1} = 2\sqrt{5} = \sqrt{20}$
$k^2 - 2k + 17 = 20$
$k^2 - 2k - 3 = 0$
$(k - 3)(k + 1) = 0$
$k = 3 \vee k = -1$
$k = 3 : 3^2 - 2 \cdot 3 + 17 = 9 - 6 + 17 = 20$
$k = -1 : (-1)^2 - 2 \cdot (-1) + 17 = 1 + 2 + 17 = 20$